Discovery Exercise for Scalar and Vector Fields

The xy plane is covered with sand. Wandering the plane in your bare feet, you conclude that the depth of sand at any given point can be modeled by the equation $z = x^2 \sin^2 y$.

1. What is the minimum depth of the sand? List three points where you would find this depth.

2. Explain why no point can claim to have the maximum depth of sand.

3. Starting at the point $(1, \pi/2)$, you take a long walk in the positive x-direction. Describe what you see (or feel), over time, happening to the sand depth under your feet.

   See Check Yourself #50 at felderbooks.com/checkyourself

4. Returning to $(1, \pi/2)$ you take another walk, this time in the positive y-direction. Describe your experience during this different walk.

Six months later the sand has been swept away by seven maids with seven mops, and has been replaced with a uniform shallow layer of water. Unlike the sand, which was stationary, this water is moving around. The velocity of the water at any point is given by $\vec{v} = x^2\hat{i} + 3\hat{j}$.

5. If you drop a leaf on the water at the point $(0, 2)$, which way will it head, and how fast? Describe its motion over time.

6. If you drop a leaf on the water at the point $(1, 2)$, which way will it head, and how fast? Describe its motion over time. Your description should include its approximate heading in the short term (soon after you drop it) and the long term (much later).

See Check Yourself #51 at felderbooks.com/checkyourself

7. If you drop a leaf on the water at the point $(-1, 2)$, which way will it head, and how fast? Describe its motion over time. Your description should include its approximate heading in the short term (soon after you drop it) and the long term (much later).