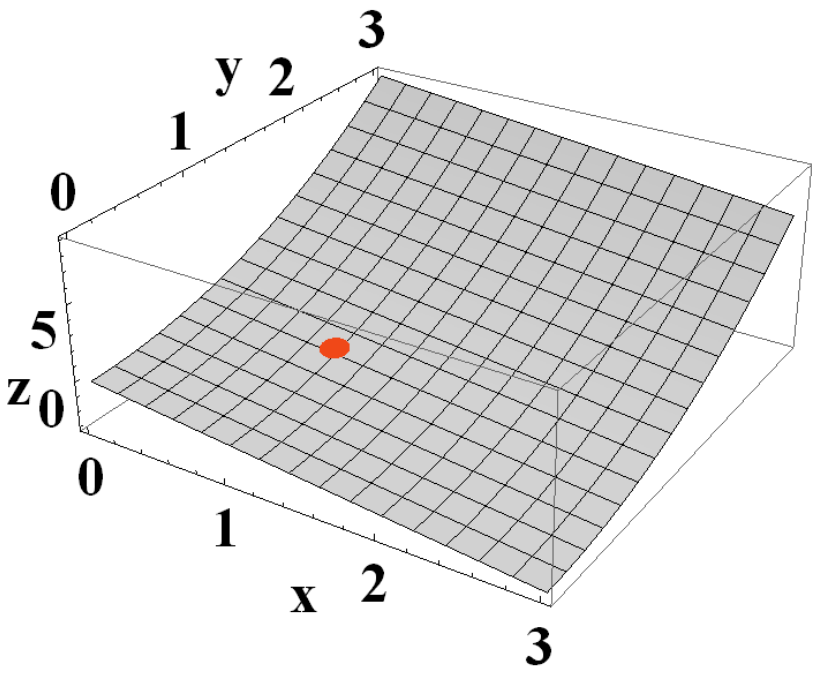
Discovery Exercise for Directional Derivatives

The following picture shows the function $z = y^2 - x$ on the domain $x \in [0, 3]$, $y \in [0, 3]$. The point $(1, 1, 0)$ is labeled.



In all the questions that follow, assume that you begin at the point $(1, 1, 0)$*. As you change your* x *and* y *coordinates, your* z *coordinate changes to keep you on the surface.*

Parts 1–4 can be answered exactly using partial derivatives. Make sure, however, that your answers make sense with the picture.

1. If you move in the positive x-direction, are you moving up or down? With what slope?

2. If you move in the negative x-direction, are you moving up or down? With what slope?

3. If you move in the positive y-direction, are you moving up or down? With what slope?

4. If you move in the negative y-direction, are you moving up or down? With what slope?

   See Check Yourself #22 at felderbooks.com/checkyourself

Parts 5–8 should be answered approximately, based on the picture.

5. Now suppose you move diagonally, following the vector $\hat{i} + \hat{j}$ along the xy plane (but allowing z to change as always so you stay on the surface). Are you moving up or down? With approximately what slope?

6. Suppose you follow the vector $\hat{i} - \hat{j}$ along the xy plane. Are you moving up or down? With approximately what slope?

7. What direction would you move along the xy plane if you wanted to go *upward* as steeply as possible? (You'll know how to calculate this easily when you learn about the "gradient.")

8. If you placed a ball on this surface at the point $(1, 1, 0)$, which way would it roll?