

Discovery Exercise for Separation of Variables

Consider the equation:

$$\frac{dy}{dx} = x^2y \quad (1)$$

Before you begin the process, you may want to see if you can figure out the general solution to this equation by thinking about it. The answer is not immediately obvious, but it does make sense if you think about it in the right way! Below we show you one way to find the answer if you can't puzzle it out.

1. Begin by dividing both sides of Equation 1 by y .
2. Then multiply both sides of the equation by dx . The variables are now *separated*: all the x -dependence is on one side, and all the y -dependence on the other.
3. Write an integral sign in front of both sides.
4. Integrate both sides. Because this is an indefinite integral, you need a $+C$ on one side or the other. Traditionally, it is put on the right (the x side).
5. Solve the resulting equation for y .

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6. Check to make sure your solution solves Equation 1.
7. The solution to this equation is more commonly written as $y = Ce^{x^3/3}$. Explain why this is the same as the previous solution, even though they look quite different.