Discovery Exercise for Arbitrary Constants

1. For the differential equation $dy/dx = -3$ the solution can be written as $y = -3x + C$.
   (a) Plug in $C = 3$ and show that the resulting function is a valid solution of the differential equation.

   (b) Plug in $C = 0$ and show that the resulting function is a valid solution of the differential equation.

   (c) Plug in $C = x$ and show that the resulting function is *not* a valid solution of the differential equation.

   (d) What sorts of C-values are guaranteed to result in valid solutions?

   (e) What is the *only* C-value that satisfies the condition $y(-4) = 15$? (To find it, let $x = -4$ and $y = 15$ and solve for C.)

2. Consider the differential equation $dy/dx = e^y$.
   (a) Which of the following functions are valid solutions? (List all that apply.)
      i. $y = e^x$
      ii. $y = \ln x$
      iii. $y = -\ln(-x)$
      iv. $y = -\ln(-x) + 4$
      v. $y = -4\ln(-x)$
      vi. $y = -\ln(-x + 4)$
      vii. $y = -\ln(-x + 7)$

   (b) Based on your answers, write a function that has a C in it, about which you can say, "This function is a valid solution to $dy/dx = e^y$ for any value of the constant C."

   (c) Confirm that your solution works for $C = -3$.

   (d) Find the value of C for which $y(0) = 0$.