Discovery Exercise for Introduction to Fourier Series

When you learned about Taylor series you learned that most normal functions can be written as sums of powers of x: $f(x) = c_0 + c_1x + c_2x^2 + \ldots$. You're going to learn to write functions as sums of sines and cosines, which turns out to be useful in different circumstances. To get you started, let's see what some simple sums of cosines look like. In the first couple of parts you'll generate your own plots, and in the next couple you'll answer questions about the plots below.

1. Sketch $\cos(x)$ and $\cos(10x)$ on the same plot. (Choose a range of x values big enough to see the behavior of each function clearly.)

2. Sketch $\cos(x) + .1\cos(10x)$. You can see how to do this by taking your $\cos(x)$ sketch and adding small wiggles to it for the $\cos(10x)$ term.

3. The left plot in Figure 1 is the sum of two different cosine functions. Estimate the amplitude and period of both cosines. To write this in the form $A\cos(p_1x) + B\cos(p_2x)$ remember that the period of $\cos(px)$ is $2\pi/p$.

   See Check Yourself #58 at felderbooks.com/checkyourself

4. Similarly, try to identify the two cosine waves that make up the right-hand plot in Figure 1.

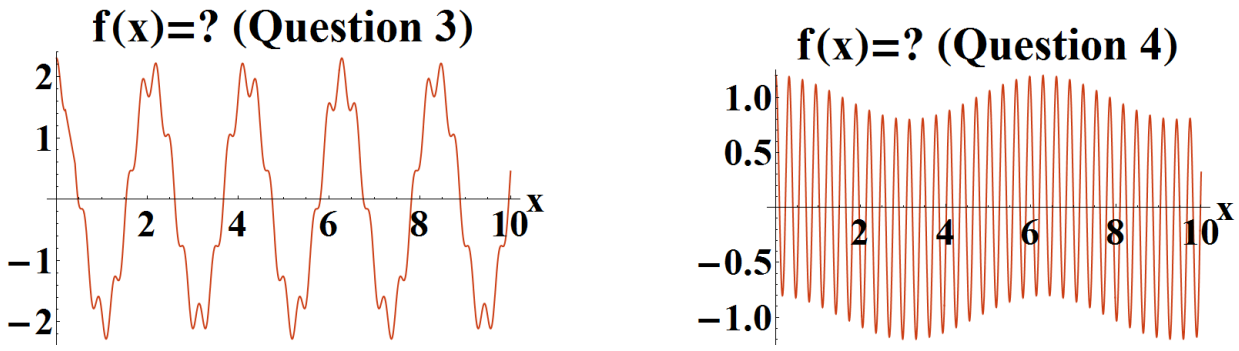

Figure 1: Each plot is a sum of two cosines. Use these in answering Parts 3 and 4.

The examples we've used so far looked fairly simple because one wave had a much higher amplitude than the other. When you add waves with similar amplitudes, the results tend to look messier.

5.  Make a plot of $\cos(5x) + \cos(6x) + \cos(7x)$ and copy it onto your paper. It's a good way to start to see that even simple combinations of cosines can lead to pretty complicated behavior.