Discovery Exercise for Reduction of Order

Consider the equation

$$x^2y''(x) - (2x^2 + x)y'(x) + (x^2 + x)y(x) = 0 \tag{1}$$

1. Confirm that $y_1 = e^x$ is a solution to this equation.

2. The second solution is harder to guess, but you can make it easier by writing it in the form $y_2 = u(x)y_1(x)$, where y_1 is the solution we just gave you and $u(x)$ is an unknown function. Plug this into the differential equation to get a differential equation for $u(x)$. Simplify your answer as much as possible.

   See Check Yourself #67 at felderbooks.com/checkyourself

3. Your equation for u should have u'' and u' in it, but not u by itself. Make the substitution $v = u'$ to get a first order differential equation for v.

4. Solve the equation you wrote for $v(x)$ and use that to find $u(x)$.

5. Write the general solution to Equation 1. Plug it in and verify that it works.

The technique you just used is called "reduction of order." When you have one solution y_1 to a linear, second order ODE , the guess $y_2 = uy_1$ will give you a *first order* ODE to solve for u'.