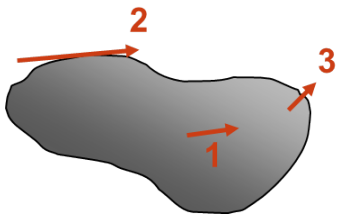
Discovery Exercise for the Divergence Theorem

The picture shows a three-dimensional region bounded by a closed surface. A vector field $\vec{v}$ is defined everywhere within this region, but we have shown three vectors in particular. Vector (1) is entirely within the region; vector (2) is at the edge of the region, and tangent to the bounding surface; vector (3) is at the edge of the region and points directly out from the bounding surface.



Your job is to add up the divergence of the vector field throughout the region: that is, you want to find $\iiint \left(\vec{\nabla} \cdot \vec{v}\right) dV$. Of course you can't do that without knowing more, but we want to focus on how *these three vectors* contribute to the total.

1. Remember that your goal is to add up the divergence *within the region only*—you don't actually care what happens outside. Given that...

   (a) Will vector (1) contribute a positive number to the total, a negative number, or zero?

   (b) Will vector (2) contribute a positive number to the total, a negative number, or zero?

   (c) Will vector (3) contribute a positive number to the total, a negative number, or zero?

2. Now, in general, what kinds of vectors will contribute to a positive total divergence within the region?