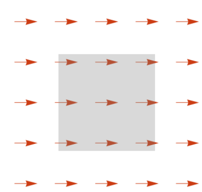
Discovery Exercise for Divergence

The vector field $\vec{V}(x,y)$ represents how much mass passes by, per unit cross-sectional area per unit time. The "divergence" of $\vec{V}$ represents how much the gas is flowing *away from* a given point. To approach the divergence, we consider a square region. As the gas flows through this region, we ask the question: Is gas flowing out of this region faster than it is flowing in? (Generally losing gas, positive divergence.) Or is gas flowing into this region faster than it is flowing out? (Generally gaining gas, negative divergence.) Or is gas flowing in and out at the same rate? (Neither gaining nor losing gas, zero divergence.)

In all our examples, the gas is flowing to the right. Assume the constant k in the equations below is positive. In our first example, the flow of the gas is constant. This field could be represented as $\vec{V}_1 = k\hat{i}$.

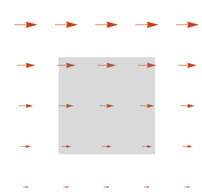


1. Compare the rates at which gas is entering our region (from the left) and exiting (to the right). Which rate is faster, or are they the same?

2. Is this region generally losing gas, gaining gas, or neither?

3. Is the divergence of $k\hat{i}$ positive, negative, or zero?

In our second example, the flow of the gas increases as you move up. This field could be represented as $\vec{V}_2 = ky\hat{i}$.



4. Compare the rates at which gas is entering our region (from the left) and exiting (to the right). Which rate is faster, or are they the same?

5. Is this region generally losing gas, gaining gas, or neither?

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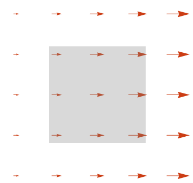
6. Is the divergence of $ky\hat{i}$ positive, negative, or zero?

In our third example, the flow of the gas increases as you move to the right. This field could be represented as $\vec{V}_3 = kx\hat{i}$.



7. Compare the rates at which gas is entering our region (from the left) and exiting (to the right). Which rate is faster, or are they the same?

8. Is this region generally losing gas, gaining gas, or neither?

9. Is the divergence of $kx\hat{i}$ positive, negative, or zero?

10. Create a vector flow where the divergence is negative. Represent your flow with both a drawing and a formula.

11. Now think about examples such as $e^x\hat{i}$, $(1/x)\hat{i}$, $(x-y)\hat{i}$, and, more generally, $f_x(x,y)\hat{i}$. For vector flows that are strictly in the x-direction (as all our examples were), what has to be true of f_x for the divergence to be positive?

12. Finally, think about two-dimensional vector fields. What has to be true of f_x and f_y for the divergence to be positive?