Computer Problems for Vector Calculus

1. The average rainfall in Flobbertown follows the strange pattern $R = (1 + \sin x)\, e^{-x^2 y^2}$, where x and y are distances north and east from one corner of the town.

   (a) Pick at least 2 values of x and sketch the rainfall function $R(y)$ at that value. Label your plots with their x values.

   (b) Pick at least 2 values of y and sketch the rainfall function $R(x)$ at that value. Label your plots with their x values.

   (c) Have a computer generate a 3D plot of $R(x, y)$. Make sure you plot over a region that clearly shows the general behavior of the function, and includes all the x and y values you used for your sketches. Check if the computer plot matches your constant-latitude and constant-longitude sketches. (If it doesn't, figure out what you did wrong.)

   (d) Based on your computer plot, if you were to start at the position $(\pi/4, 1)$ roughly what direction could you move in to keep R constant? *Hint*: You may find it easier to answer this if you make a second plot that zooms in on a small region around this point.

2. The town of Chewandswallow has been buried in piles of bread. The depth of bread is given by $B = \cos(x + y) + \sin\left(x^2 + y^2\right)$, where the town covers the region $0 \le x \le \pi/2$, $0 \le y \le \pi$.

   (a) Have a computer make a contour plot of $B(x, y)$.

   (b) Describe the shape of the contours.

   (c) Based on your plot, if you were in the center of town and wanted to slide down a mountain of bread, in approximately what direction would you get the steepest descent?

3. An object whose mass is extended from $x = 0$ to $x = L$ has density $\lambda = kx$ where k is a positive constant.

   (a) Set up and evaluate an integral to calculate the gravitational potential created by this object at a point $x > L$, using the rule $V = -GM/x$.

   (b) Show that your answer has the correct units for potential. (In order to do this, you will have to first figure out the units for λ.)

   (c) Set up and evaluate an integral to calculate the gravitational field created by this object at a point $x > L$, using the rule $g = -GM/x^2$.

   (d) Confirm that your potential and gravitational fields have the correct relationship to each other.

4. For each of the functions below, make a 3D plot and a contour plot in the region $-1 \le x \le 1$, $-1 \le y \le 1$. On the contour plot show at least three arrows representing the gradient at different points.

   (a) $f(x, y) = e^{-(x-1)^2 - y^2} + e^{-(x+1)^2 - y^2}$

   (b) $f(x, y) = \sin(xy)$

   (c) $f(x, y) = \sin(x^2 y)$

In Problems 5–6. . .

(a) Find the potential function F for the vector field $\vec{v}$. As always, you should do that by choosing the easiest line integral you can from an $F = 0$ point to (x, y, z).

(b) Evaluate the given line integral by using the gradient theorem, based on the potential function you found.

(c) Evaluate the given line integral again, this time *not* using the gradient theorem, and confirm that you get the same answer.

5. $\vec{v} = 2x\sin(3z)e^{x^2+y}\hat{i} + \sin(3z)e^{x^2+y}\hat{j} + 3\cos(3z)e^{x^2+y}\hat{k}$ along the curve $x = 2t$, $y = e^t$, $z = t^2$ as t goes from 0 to 1. *The integral to find F can be done easily by hand, but the line integral will require a computer or integral table.*

6. $\vec{v} = (x+1)e^{x+y}\hat{i} + xe^{x+y}\hat{j}$ along the curve $x = \cos t$, $y = \sin t$ from $(1, 0)$ to $(0, 1)$. *The integral to find F can be done with integration by parts or on a computer, but the line integral will require a computer or integral table.*

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7. $f(x,y) = e^{-(x-1)^2-y^2} - e^{-(x+1)^2-y^2}$

   (a) Have a computer make a 3D plot of $f(x, y)$. Choose a domain for your plot that lets you clearly see how the function behaves.

   (b) Based on your plot, predict the sign of the Laplacian at the points $(-1, 0)$, $(0, 0)$, and $(1, 0)$. For each one explain why you would expect the answer you gave based on the plot.

   (c) Calculate $\nabla^2 f$ (by hand or on a computer) and check your predictions.