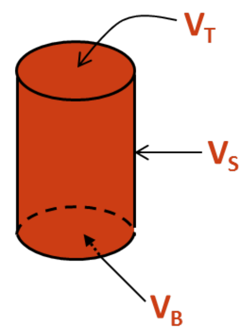
Discovery Exercise for Separation of Variables—Inhomogeneous Boundary Conditions

1. Separation of variables is simplest when a problem has only one inhomogeneous boundary *or* initial condition. Explain why the technique of separation of variables, in its simplest form, relies on this limitation.

Now consider a problem with three inhomogeneous boundary conditions. The cylinder below has no charge inside. The potential therefore obeys Laplace's equation, which in cylindrical coordinates is:

$$\frac{\partial^2 V}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial V}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$



The potential on top is given by V_T, which could be a constant or a function of ρ or even a function of ρ and ϕ. (For our present purpose it doesn't really matter.) The potential on the bottom is given by V_B. The potential on the side is V_S which could be a function of both z and ϕ.

2. The approach to such a problem is to begin by solving three *different* problems. Each problem is the same differential equation as the original, but each has only one inhomogeneous boundary condition. In the first such sub-problem, we take $V = V_T$ on top, but $V = 0$ on the side and bottom. What are the other two sub-problems?

3. If you solved all three sub-problems and added the solutions, would the resulting function solve the original differential equation? Would it solve all the original boundary conditions?