Discovery Exercise for Partial Derivatives

The drawing below shows a function $z(x, y)$, with one point on the plot marked.

1. If you start at the marked point and move in the positive $x$ direction, holding $y$ constant, is $z$ increasing, decreasing, or staying constant?

2. If you start at the marked point and move in the positive $y$ direction, holding $x$ constant, is $z$ increasing, decreasing, or staying constant?

The rate of change of $z$ in the $x$ direction, holding $y$ constant, is “the derivative of $z$ with respect to $x$,” usually written $\partial z/\partial x$.

3. Based on your answers above, is $\partial z/\partial x$ positive, negative, or zero at the marked point?

   See Check Yourself #19 at felderbooks.com/checkyourself

4. What about $\partial z/\partial y$?

5. Looking at the plot, is $\partial^2 z/\partial x^2$ at the marked point positive, negative, or zero? Explain what about the surface lets you know.

6. Suppose that $z$ represents the concentration of salt in a lake, $y$ represents depth in that lake, and $x$ represents time. Explain what each of your answers to Parts 3–5 tells you physically about the lake.