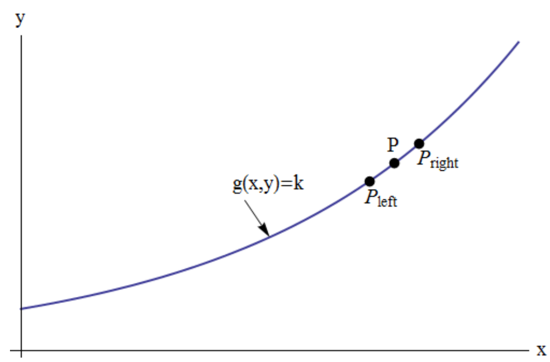
Discovery Exercise for Lagrange Multipliers

There are two functions in this exercise, and it's important not to confuse them.

- The "objective" function, $f(x, y)$, is the one we really care about. This function is not shown in the drawing below.
- The other function will be called here $g(x, y)$. When we set this function equal to a constant we get the curve $g(x, y) = k$ shown below. That curve is our "constraint."

We are interested here in values of the function $f(x, y)$, but only along the curve defined by $g(x, y) = k$. Specifically we are interested in finding the maximum value that $f(x, y)$ attains along that curve. Note that this may not correspond to a local maximum of the function $f(x, y)$.



The drawing shows the curve $g(x, y) = k$ and three points P_{left}, P, and P_{right} on that curve. The vector $\vec{v}$ points parallel to the curve at position P, generally in the direction of P_{right}.

1. For this part only, suppose that $D_{\vec{v}}f$ at point P is positive.

   (a) As you move from P toward P_{right} does the value of $f(x, y)$ increase, decrease, or stay the same?

   (b) As you move from P toward P_{left} does the value of $f(x, y)$ increase, decrease, or stay the same?

2. Now, for this part only, suppose that $D_{\vec{v}}f$ at point P is negative. Explain how we know that point P cannot possibly represent the maximum value of f along the curve.

For the remaining questions in this exercise, suppose that point P does in fact represent the maximum value of f along the curve.

3. What does that assumption imply about $D_{\vec{v}}f$ at point P? Explain briefly how you know.

4. What does your answer to Part 3 imply about the gradient $\vec{\nabla}f$ at point P? Explain briefly how you know. *Hint:* it doesn't imply $\vec{\nabla}f = 0$.

5. Which way does $\vec{\nabla}g$ point at point P? Explain briefly how you know.

6. Use your answers to Parts 4–5 to write an equation relating $\vec{\nabla}f$ and $\vec{\nabla}g$ at the point P where f takes on its maximum along the curve $g = k$.