

Discovery Exercise for Legendre Polynomials

Consider the following differential equation where μ is a constant.

$$(1 - x^2)y'' - 2xy' + \mu y = 0 \quad -1 < x < 1 \quad (1)$$

1. Assuming a power series solution of the form $y(x) = \sum c_n x^n$, find the recurrence relation for c_{n+2} in terms of c_n , n , and the constant μ .

See Check Yourself #85 at felderbooks.com/checkyourself

Now consider the particular solution for the constants $\mu = 20$, $c_0 = -3$, and $c_1 = 0$.

2. Show that all the odd coefficients will be zero.
3. Find c_2 , c_4 , and so on until you reach a point where all subsequent even coefficients will be zero.
4. Write down the resulting polynomial and verify that it solves Equation 1 for $\mu = 20$.