

Discovery Exercise for Linearly Independent Solutions and the Wronskian

1. Each of the following functions is a valid solution to the differential equation $y''(x) + k^2y(x) = 0$. But four of these functions represent (in different forms) the *general solution*, and the other three do not. Which ones?
 - (a) $y(x) = \sin(kx)$
 - (b) $y(x) = A \sin(kx) + B \cos(kx)$
 - (c) $y(x) = A \sin(kx + \phi)$
 - (d) $y(x) = e^{ikx}$
 - (e) $y(x) = Ae^{ikx} + B \sin(kx)$
 - (f) $y(x) = Ae^{ikx} + Be^{-ikx}$
 - (g) $y(x) = Ae^{ikx+\phi}$

See Check Yourself #64 at felderbooks.com/checkyourself

2. Each of the following functions is a valid solution to the differential equation $4x^2y''(x) + y(x) = 0$. But two of these functions represent (in different forms) the *general solution*, and the other three do not. Which ones?
 - (a) $y(x) = \sqrt{x}$
 - (b) $y(x) = \sqrt{x} \ln x$
 - (c) $y(x) = A\sqrt{x} + B\sqrt{x} \ln x$
 - (d) $y(x) = A\sqrt{x} + B\sqrt{x}(1 + \ln x)$
 - (e) $y(x) = (A + B)\sqrt{x} \ln x$