

Discovery Exercise for Variable Substitution

Consider the differential equation $dx/dt = (x+t)^2 - 1$. It's not linear, it's not separable, it's not exact, and it doesn't lend itself to any obvious guess. The trick to solving it is to rewrite it using different variables. The process is a generalization of what you do when you evaluate an integral using u -substitution. We're going to use the substitution that most readily suggests itself from this equation: $u = x + t$.

1. Recall that u and x are both functions of t . Take the derivative of both sides of $u = x + t$ to get an equation relating du/dt and dx/dt .

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2. Use $u = x + t$ and the equation you found for du/dt to rewrite the original differential equation in terms of u and t , with no x in it.

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3. Solve the resulting equation for $u(t)$ by separation of variables.
4. Write the general solution for $x(t)$. Your solution should include an arbitrary constant, and should not include the letter u .
5. Verify that your solution solves the original differential equation.