Discovery Exercise for Guess and Check

A linear second-order differential equation starts with the following expression:

\[ a_2(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x)y \]  

(1)

Suppose the three different functions \( c_1(x) \), \( c_2(x) \), and \( p(x) \) have the following properties: if you plug \( y = c_1(x) \) into Equation 1 you get 0. If you plug in \( y = c_2(x) \) in, you again get 0. And if you plug in \( y = p(x) \), you get out the function \( A(x) \).

1. What do you get if you plug \( y = 3c_1(x) \) into Equation 1?
2. What do you get if you plug \( y = 5c_2(x) \) into Equation 1?
3. What do you get if you plug \( y = 10p(x) \) into Equation 1?
4. What do you get if you plug \( y = 13c_1(x) - 12c_2(x) + p(x) \) into Equation 1?

See Check Yourself #63 at felderbooks.com/checkyourself

5. Would your answers change if Equation 1 began with \( a_3(x)y'''(x) \)? (Assume that the functions \( c_1(x) \), \( c_2(x) \), and \( p(x) \) still gave the same answers as before.)

6. Would your answers change if Equation 1 contained \( a_0(x)y^2 \) instead of \( a_0(x)y \)? (same comment)