# Discovery Exercise for Green's Functions

The following differential equation suggests Laplace transforms—and you can solve it that way and get the right answer—but here you're going to use a different approach.

$$\frac{dx}{dt} + x = \delta(t-3) \quad \text{with} \quad x(0) = 0$$

1. For all $t \neq 3$ this equation is just $\dot{x} + x = 0$. Write the general solution to this equation.

2. For $t < 3$ you have the initial condition. Plug it in to find the arbitrary constant and write the solution.

3. At $t = 3$ the function undergoes a discontinuous but finite jump upward. After $t = 3$ your solution from Part 1 applies again, although with a different value of the arbitrary constant. Based on all that information, sketch the solution for $t \geq 0$. Your sketch does not have to be accurate about the size of the $t = 3$ jump, but should be qualitatively correct in all other respects.

*You can check yourself by solving the same equation with a Laplace transform!*