Discovery Exercise for Overview of Ordinary Differential Equations

1. A certain function \( y(x) \) has the following property: its derivative is \( 6x \). We can express this mathematically by writing \( \frac{dy}{dx} = 6x \).
   
   (a) Solve this equation; that is, find a function \( y(x) \) that works.
   
   (b) Find a different function that also works. \((\text{Hint: It will only be slightly different from the first function.})\)

   \[ \text{See Check Yourself \#3 at felderbooks.com/checkyourself} \]
   
   (c) Test your solutions by taking their derivatives and making sure you get \( 6x \) both times.

2. A certain function has the following property: its second derivative is \( 6x \).
   
   (a) Write an equation that expresses the sentence “The second derivative of the function \( y \) is \( 6x \).”
   
   (b) Solve the equation; that is, find a function \( y \) that works.
   
   (c) Find a different function that also works.
   
   (d) Test your solutions.

3. A certain function has the following property: the function is its own derivative.
   
   (a) Write an equation that expresses the sentence “Function \( y \) is its own derivative.”
   
   \[ \text{See Check Yourself \#4 at felderbooks.com/checkyourself} \]
   
   (b) Solve the equation; that is, find a function \( y \) that works.
   
   (c) Test your solution by taking its derivative and making sure you get the same function you started with.
   
   (d) Find a different function that also works and test it. \((\text{Hint: adding a constant to your first solution will not work!})\)
4. A certain function has the following property: *when you take the derivative of the function, you get the same function back, times 2.*

(a) Write an equation that expresses the sentence “When you take the derivative of function $y$, you get the original function $y$ times 2.”

(b) Solve the equation; that is, find a function $y$ that works.

(c) Find a *different* function that also works.

(d) Test your solutions.

5. A certain function $y(x)$ is the solution to the equation $dy/dx = xy^2$.

(a) Express this equation in words. (This is the reverse of what you were doing before, where we gave you the words and you gave us the equation.)

(b) Which of the following functions could be $y(x)$? *Hint:* the only way you can approach this is to try each function in that equation. One function will work, the others will not. For the left side of the equation, find $dy/dx$. For the right side, square $y$ and then multiply the answer by $x$. If the two sides come out the same, you have found a solution!
   i. $y = \sqrt{x^2 + 9}$
   ii. $y = e^{\sqrt{x}+3}$
   iii. $y = -2/(x^2 + 3)$
   iv. $y = x^3/3 + 6$