Discovery Exercise for Coupled Equations

Consider a population of foxes and rabbits. They each reproduce, but their populations are both limited by the fact that foxes eat rabbits. For this exercise we'll adopt a simplified model of this relationship.

1. Write a differential equation for the rabbit population $R(t)$ that expresses the sentence: "Each year every rabbit produces 5 babies on average, but 10 rabbits are killed by each fox." Use $F(t)$ for the number of foxes.

   See Check Yourself #7 at felderbooks.com/checkyourself

2. Explain why you cannot solve this equation for $R(t)$ with the information given. What more information would you need? (*Hint*: The answer is not the initial number $R(0)$. Not knowing that just means there would be an arbitrary constant in your answer.)

3. In Question 1 we gave you a verbal description of the rabbit population and asked you for the differential equation. Here we will do the opposite for the foxes. The fox population is described by the differential equation $dF/dt = R/2 - F$. Give the verbal description that explains where this equation comes from.

4. If $dR/dt = dF/dt = 0$ then both populations remain constant. What would have to be true about the values of R and F for this condition to hold? (They would not both have to be zero, although that is one way to get this result.)

5. Which of the following pairs of functions solve the equations for R and F? (More than one answer may be correct. Indicate all of the correct solutions.)

   (a) $R(t) = 2000$, $F(t) = 1000$.

   (b) $R(t) = 2\cos t$, $F(t) = 2\cos t$

   (c) $R(t) = 1000e^{4t} + 200$, $F(t) = 100e^{4t} + 100$

   (d) $R(t) = e^{4t}$, $F(t) = 2e^{4t}$