Chapter: Ordinary Differential Equations

Discovery Exercise for Arbitrary Constants

1. For the differential equation \( \frac{dy}{dx} = -3 \) the solution can be written as \( y = -3x + C \).

   (a) Plug in \( C = 3 \) and show that the resulting function is a valid solution of the differential equation.

   (b) Plug in \( C = 0 \) and show that the resulting function is a valid solution of the differential equation.

   (c) Plug in \( C = x \) and show that the resulting function is not a valid solution of the differential equation.

   (d) What sorts of \( C \)-values are guaranteed to result in valid solutions?

   (e) What is the only \( C \)-value that satisfies the condition \( y(-4) = 15 \)? (To find it, let \( x = -4 \) and \( y = 15 \) and solve for \( C \).)

2. Consider the differential equation \( \frac{dy}{dx} = e^y \).

   (a) Which of the following functions are valid solutions? (List all that apply.)

   \[ 
   \begin{align*}
   &i. \ y = e^x \\
   &ii. \ y = \ln x \\
   &iii. \ y = -\ln(-x) \\
   &iv. \ y = -\ln(-x) + 4 \\
   &v. \ y = -4\ln(-x) \\
   &vi. \ y = -\ln(-x + 4) \\
   &vii. \ y = -\ln(-x + 7)
   \end{align*} 
   \]

   (b) Based on your answers, write a function that has a \( C \) in it, about which you can say, “This function is a valid solution to \( \frac{dy}{dx} = e^y \) for any value of the constant \( C \).”

   (c) Confirm that your solution works for \( C = -3 \).

   (d) Find the value of \( C \) for which \( y(0) = 0 \).