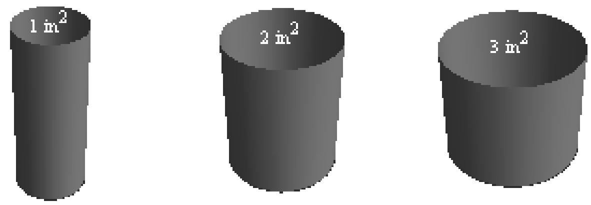
Discovery Exercise for Surface Integrals

Rainfall is often measured in inches per hour. If that seems strange, imagine placing these three buckets side by side in a rainstorm. The rising waterline in each bucket, measured in inches per hour, will be exactly the same.



1. Although (as stated above) the three buckets will see the same rising waterline, their total accumulation of water (measured in gallons/hour for instance, or in^3/hour) will be very different. Which of the following is primarily responsible for this difference? (Assume none of the buckets fills up.)
   (a) The tops, through which the rain enters, have different surface areas.
   (b) The bottoms, on which the rain lands, have different surface areas.
   (c) The buckets are different heights.
   (d) The buckets have different volumes.

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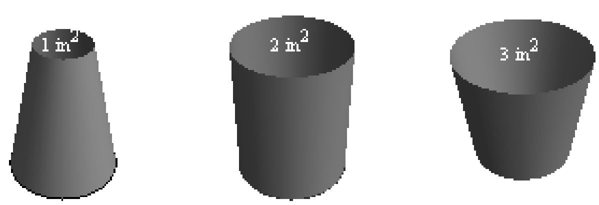
In the three buckets below, the accumulation will again be different. We are interested in measuring the total accumulation of water in in^3/hour.



2. A rainfall of 0.2 inches per hour is considered "moderate." After one hour of such a rain, the middle bucket shown above will have a waterline at 0.2 inches. How much water (measured in in^3) will it have accumulated?

3. After one hour of the same rain, how much water (measured in in^3) will each of the *other* two buckets accumulated? *Hint:* "accumulated water" is the same as "total amount of water that has passed through the top."

4. A rainfall of 0.4 inches per hour is considered "heavy." After one hour of such a rain, how much water has accumulated in each bucket?

5. Write a formula for the total amount of water that accumulates in a bucket after one hour. Your formula should have two independent variables.

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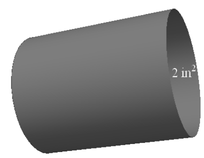
Now we're going to change the rate of accumulation *without* changing either the rate of the rainfall, or the area of the top of the bucket. How? Tilt the bucket!



6. How fast does the water accumulate if we tilt the bucket 90° from its starting position?

7. Let r equal the rate of rainfall, A the area of the top of the bucket, and θ the angle of tilt, going from 0 in the original vertical position to $\pi/2$ in a completely horizontal position. Which of the following is a reasonable formula for the rate of water accumulation W?

   (a) $W = rA$
   (b) $W = rA\theta$
   (c) $W = rA\cos\theta$
   (d) $W = rA\sin\theta$
   (e) $W = rA\tan\theta$
   (f) none of the above is reasonable