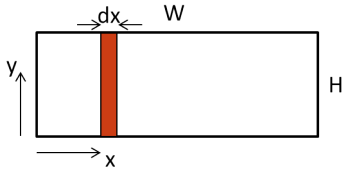
Discovery Exercise for Cartesian Double Integrals over a Rectangular Region

1. A horizontal plank with height H and width W has a density given by $\sigma = kx$, where x is the distance from the left side of the strip and k is a constant. You want to calculate the mass of the plank.

   (a) You begin by drawing a thin vertical strip on your box, as shown below. What is the area dA of this thin strip?



   (b) This strip is at position x in the horizontal direction. Multiply the area of the strip times its density to find the mass dm of this thin strip.

   (c) Put an integral sign in front of the expression you just wrote for dm and fill in the appropriate limits of integration. Evaluate this integral to find the mass M of the plank. You should be able to find the units of k from the expression $\sigma = kx$ and use them to check that your answer has units of mass. (Remember that density for a 2D object has units of mass per area.)

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   (d) Explain why the procedure you just followed would not have worked if you had started by drawing a thin horizontal strip instead of a vertical one.

Now you will redo this problem assuming the density is given by $\sigma = qxy$. (You should figure out the units of the constant q so you will be able to check the units on your final answer.) Once again you begin by drawing a thin strip as shown in the picture above. Its area dA is the same as what you calculated above.

2. Explain why you cannot find the mass of the strip dm just by multiplying density times area as you did above.

3. Supposing our thin strip were drawn at $x = 1$, set up and evaluate an integral with respect to y for the mass of the strip. Your integral will involve dx and dy and q, but after you integrate with respect to y your final answer will be a function of dx and q.

4. Now supposing our thin strip were drawn at $x = 2$, set up and evaluate an integral with respect to y for the mass of the strip.

5. Now generalize: our thin strip is drawn at some fixed x value. Set up and evaluate an integral with respect to y for the mass of the strip. Your final answer will be a function of x, dx, and q.

6. Put an integral sign in front of the expression you just wrote for dm and fill in the appropriate limits of integration. Evaluate this integral to find the mass M of the plank (and check that your answer has correct units).

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7. In Part 1 you calculated the mass of a two-dimensional object using only one integral. What was it about the second problem that made it require two integrals?