Discovery Exercise for the Identity and Inverse Matrices

For the purposes of this exercise, let $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$.

1. Find a matrix $\mathbf{I}$ such that $\mathbf{AI} = \mathbf{A}$; that is, when you multiply our $\mathbf{A}$ by your $\mathbf{I}$, the result is our matrix $\mathbf{A}$ again. (We haven't given you any method for doing this, but you can probably get there pretty quickly by trial and error. Don't go farther until you confirm that your answer works!)

2. We know that matrix multiplication is not generally "commutative"; that is, $\mathbf{AB} \neq \mathbf{BA}$. Show that the multiplication in Part 1 *is* commutative, meaning $\mathbf{AI} = \mathbf{IA}$.

3. Your goal is now to find a matrix $\mathbf{B}$ such that $\mathbf{AB} = \mathbf{I}$ (where $\mathbf{I}$ is the matrix you found in Question 1).

   (a) Explain why matrix $\mathbf{B}$ must have dimensions 2×2.

   Now that we know the dimensions, we will let $\mathbf{B} = \begin{pmatrix} w & y \\ x & z \end{pmatrix}$; in other words, use letters to represent all the unknown numbers. The defining equation $\mathbf{AB} = \mathbf{I}$ therefore becomes:

$$\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} w & y \\ x & z \end{pmatrix} = \mathbf{I} \tag{1}$$

   (b) Perform the matrix multiplication on the left side of Equation 1. The result should be another 2×2 matrix that you are setting equal to $\mathbf{I}$.

(c) Recall that if two matrices equal each other, all the corresponding elements must be equal. Use this fact to write four equations to solve for the unknown w, x, y, and z.

(d) Solve your equations to find matrix $\mathbf{B}$.

See Check Yourself #41 at felderbooks.com/checkyourself

4. Show that the multiplication in Part 3 is commutative, meaning $\mathbf{AB} = \mathbf{BA}$.