Discovery Exercise for the Easy Matrix Stuff

The following matrix, stolen from a rusted lockbox in the back of a large, dark lecture hall, is the gradebook for Professor Snape's class in potions.

$$\mathbf{G} = \begin{array}{c} \\ \textit{Granger, H} \\ \textit{Longbottom, N} \\ \textit{Malfoy, D} \\ \textit{Potter, H} \\ \textit{Weasley, R} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Poison} & \textit{Cure} & \textit{Love Philter} & \textit{Invulnerability} \end{array} \\ \begin{pmatrix} 100 & 105 & 99 & 100 \\ 80 & 90 & 85 & 85 \\ 95 & 90 & 10 & 85 \\ 70 & 75 & 70 & 75 \\ 85 & 90 & 95 & 90 \end{pmatrix} \end{array}$$

When we say this is a "matrix" we're referring to the grid of numbers. The labels (such as "Granger, H" or "Poison") explain what this particular matrix represents, but they are not part of the matrix itself.

The matrix can be viewed as a list of horizontal *rows* or as a list of vertical *columns.*

1. Is "all Longbottom's grades" a row or a column?

2. Is "all grades on Invulnerability" a row or a column?

The "dimensions" of a matrix are the number of rows and columns... in that order. So a 10×20 matrix means 10 rows and 20 columns.

3. What are the dimensions of the gradebook matrix?

For two matrices to be "equal" they must be exactly the same in every way: same dimensions, and every element the same. If everything is not precisely the same, the two matrices are not equal.

4. What must x and y be, in order to make the following matrix equal to Professor Snape's gradebook matrix?
$$\begin{pmatrix} 100 & 105 & 99 & 100 \\ 80 & x+y & 85 & 85 \\ 95 & 90 & 10 & 85 \\ 70 & 75 & x-y & 75 \\ 85 & 90 & 95 & 90 \end{pmatrix}$$

If two matrices do not have exactly the same dimensions, you cannot add or subtract them. If they do have the same dimensions, you add and subtract them just by adding or subtracting each individual element.

As an example: Professor Snape has decided that his grades are too high, so he decides to subtract the following grade-curving matrix from his original grade matrix.

$$\begin{pmatrix} 5 & 0 & 10 & 0 \\ 5 & 0 & 10 & 0 \\ 5 & 0 & 10 & 0 \\ 5 & 0 & 10 & 0 \\ 5 & 0 & 10 & 0 \end{pmatrix}$$

5. Write the new gradebook after the curve.

6. Now, suppose the professor wants to curve everyone's grades down (as before), but curve the "Potter, H" grades down twice as much. What grade-curving matrix would he subtract from his original gradebook?

Multiplying something by 3 always means adding it to itself three times, so once you know how to add matrices, you can figure out how to multiply a matrix by a number.

7. If we call the gradebook matrix $\mathbf{G}$, what is matrix $3\mathbf{G}$? In other words, what is $\mathbf{G} + \mathbf{G} + \mathbf{G}$?