Discovery Exercise for Fourier Transforms

What happens to the frequencies of a Fourier series as the period of the function gets longer? Consider a specific example. If $f(x) = e^{-x^2}$ from $-\pi$ to π, repeated periodically thereafter, the frequencies in its Fourier series are 1, 2, 3, ...

1. If we instead define $f(x) = e^{-x^2}$ from -2π to 2π, repeated periodically thereafter, what are the frequencies in its Fourier series?

2. We can get a series that models e^{-x^2} well over a wide range of values if we define $f(x) = e^{-x^2}$ from -500 to 500, and of course make it periodic outside of that. Now what are the frequencies in the Fourier series?

   See Check Yourself #61 at felderbooks.com/checkyourself

3. For each of the three functions you considered above, how many Fourier modes were there with frequencies between 0 and 10 (inclusive)? (Don't forget to include negative values of n.)

4. In the limit where you consider the entire function e^{-x^2} from $-\infty$ to ∞, what frequencies do you think you would need?

Now let's consider the amplitudes of the Fourier modes.

5. Calculate the coefficients c_0–c_3 of the complex exponential Fourier series for each of the three functions you considered above, with periods 2π, 4π, and 1000. (If you don't have access to a computer while you're doing this exercise, skip this part and the next one and at the end make a prediction as to what you would have seen.)

6. In the limit where you consider the entire function e^{-x^2} from $-\infty$ to ∞, what happens to the amplitudes c_n?

7. Given your answer to Part 3, explain why your answer to Part 6 makes sense.