Discovery Exercise for Fourier Series With Different Periods and Finite Domains

1. Consider the function \( f(x) = \sin(x) + \sin(3x) + \sin(10x) \).
   
   (a) Find \( f(\pi/6) \), \( f(13\pi/6) \), and \( f(25\pi/6) \).

   (b) Find another \( x \)-value for which \( f(x) = f(\pi/6) \).

   (c) What is the period of this function?

2. The function \( g(x) = \cos(px) \) has the property that \( g(x) = g(x + 5) \) for all \( x \) values.
   
   (a) The function \( g(x) \) might have a period of 5. What value of \( p \) would lead to this period?

   (b) The function \( g(x) \) might also have a period of 5/2. What value of \( p \) would lead to this period?

   See Check Yourself #59 at felderbooks.com/checkyourself
   
   (c) What are all the possible periods of \( g(x) \), and their corresponding \( p \)-values?

   (d) If you sum all the functions in Part (c), what is the period of the resulting function?