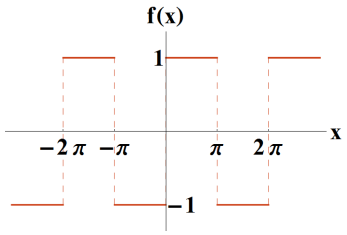
Discovery Exercise for Fourier Series with Complex Exponentials

"Square waves" are frequently used in electronics and signal processing. An example is shown below, along with its Fourier series.



$$f(x) = \frac{4}{\pi}\sin(x) + \frac{4}{3\pi}\sin(3x) + \frac{4}{5\pi}\sin(5x) + \frac{4}{7\pi}\sin(7x) + \ldots$$

Anything that can be expressed with sines and cosines can also be expressed with complex exponential functions. In this exercise you will find the complex exponential series that represents this square wave.

1. Starting with Euler's equations $e^{ix} = \cos x + i\sin x$ and $e^{-ix} = \cos x - i\sin x$, derive a formula for $\sin x$ in terms of e^{ix} and e^{-ix}.

2. Using that formula, replace all the sines in the above expression with complex exponentials.

3. Regroup your answer to find the coefficients in the following series for our square wave.

$$f(x) = c_0 + c_1 e^{ix} + c_{-1}e^{-ix} + c_2 e^{2ix} + c_{-2}e^{-2ix} + c_3 e^{3ix} + c_{-3}e^{-3ix} + \ldots$$

   (a) What is c_0?

   (b) What is c_1?

   (c) What is c_{-3}?

   See Check Yourself #60 at felderbooks.com/checkyourself

   (d) What is c_n for any even integer n?

   (e) What is c_n for any odd integer n?