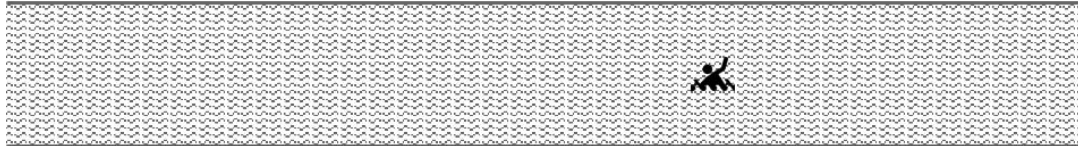


Motivating Exercise: Rescuing the Swimmer

You are working as a river lifeguard when you see a man starting to drown.



In each of the scenarios below, the question is the same: draw the path that will take you most quickly to the drowning man. In the first scenario you can draw the path exactly. In the others, your goal is a rough qualitative sketch.

- You move at the same speed in water as you do on land.
- Your speed on land is twice as fast as your speed through the water.
- The water gets deeper as you get farther from shore, and you are walking the whole way. So you enter the water at the same speed you had on land, but the farther you get from shore, the slower you walk.

We wish to call your attention to several features of this problem. It is an optimization problem, in which you are trying to minimize a particular quantity (time). But especially in the third scenario, you are not minimizing time by finding some other numerical variable (as in traditional optimization problems); you are finding a *function* (a curve) that minimizes the time. Problem of the form “find the function that minimizes this integral” will occupy this entire chapter.