APPENDIX M

Answers to Odd-Numbered Problems

Chapter 1

1.1(a) $e^{\sin x} \times \cos x$
1.3 (b), (c)
1.5 (b), (d), (e)
1.7 (a), (c), (e)
1.9 All of them work
1.11 (a), (d), (e)
1.13 (d)
1.15 (c)
1.17 (a) I. (b) V. (c) II. (d) III. (e) IV.
1.19(a) $d^2x/dt^2 = -g$
(b) $m(d^2x/dt^2) = -k|x|/x^3$
(c) **(ii)** $m(d^2x/dt^2) = k - b(dx/dt)$
(d) $m(d^2x/dt^2) = (b - rt) - k$
1.21(a) $dM/dt = 30 + M/10$
(c) D is in dollars, k in years
1.23(d) $dM/dt = S_0 1.05^t - E_0 1.02^t + .04M$
1.25(a) **(ii)** negative
(b) $dm/dt = -k^2 m^{2/3}$
(d) **(ii)** 125
1.27(a) 0
(b) -4
(c) 2
1.29(a) $Q = 0$ and $Q = 5$
1.31(a) **(ii)** e^t or e^{-t}
(b) **(ii)** $\cos t$ or $\sin t$
(c) **(ii)** e^{3t} or e^{-3t}
(d) **(ii)** $x = \cos(3t)$ or $x = \sin(3t)$
1.33(b) 2π
(c) **(ii)** phase
(d) **(ii)** $x = \sin(2t)$ or $x = \cos(2t)$, Period $= \pi$
(e) **(ii)** $2\pi/\omega$
(f) **(ii)** $\pi/5$ seconds
1.35(a) $M(t) = (\$1.05)^t M_0$
(b) $M(1) = (\$1.005)^{10} M_0$, $M(t) = (\$1.005)^{10t} M_0$
(c) **(ii)** $M = M_0 e^{.05t}$
(e) **(ii)** $R = 15e^{10t}$
1.37(a) $-e^{-x}$
(e) **(ii)** $y = 3$
(f) **(ii)** $y = -2e^{-x} + 3$

1.39(b) **(ii)** $P(t) = 7e^{-10t}$
1.41 $y = -1/(2e^{\sqrt{x}} - 11/5)$
1.43 $y = (x - 3/2)(x^2 + 1)$
1.45 (b) $u(t) = t^3 + 4e^{2t} + e^{-2t}$
1.47(d) $C = 0$
(e) $C = (1/2)\ln 5$
1.49(b) **(ii)** $f(z) = 4e^{z/2} + 5ze^{z/2}$
1.51(a) $dx/dt = k$
(b) $x = kt + C$
1.53(a) **(ii)** $dS/dt = -10$
(b) **(ii)** $S(t) = -10t + 30$
(c) **(ii)** $dS/dt = -S/3$
(d) **(ii)** $S(t) = 30e^{-(1/3)t}$
1.55(a) $dS/dt = 2 - .02S$
(c) $S(t) = Ce^{-.02t} + 100$
(d) **(ii)** $S(t) = 100(e^{-.02t} + 1)$
(e) **(ii)** $\lim_{t\to\infty} S(t) = 100$
1.57(a) **(ii)** $A = 1/2$ and $B = -1/2$
(b) **(ii)** $C = 1$ and $D = 0$
1.59(a) $dx/dt = 55$
(b) $x(t) = 55t + C$
(c) **(ii)** $x(t) = 55t - 5$
1.61(a) **(ii)** positive
(b) **(ii)** kg/s
(c) **(ii)** $m(dv/dt) = -bv$
(d) **(ii)** $v(t) = C_1 e^{-(b/m)t}$
(e) $dx/dt = C_1 e^{-(b/m)t}$
(f) $x(t) = -(m/b)C_1 e^{-(b/m)t} + C_2$
(g) **(ii)** $x(t) = (m/b)v_0(1 - e^{-(b/m)t}) + x_0$
1.63 $C = \sqrt{x_0^2 + v_0^2/\omega^2}$ and $\phi = \tan^{-1}\left[-v_0/(\omega x_0)\right]$.
1.65(e) 0
1.73 (c) $y = -2$ stable
1.75 (c) no equilibrium solutions
1.79(e) Unstable
(f) Stable
1.81(a) $m = mx + b - 2x$
(b) $2x + m = mx + b$
(c) $m = 2$ and $b = m = 2$
1.83(b) $S = 0$ and $S = P$
(d) P
1.85(a) $dV/dt = 3V - 12000$
(c) $V = 4000$
(d) **(ii)** unstable
1.87(a) **(ii)** $dQ/dt = k(Q_r - Q)$
(c) $dQ/dt = t + 20 - Q$
(e) **(ii)** $Q = t + 19$
1.89 **(ii)** $y = x^2 + C$
1.91 **(ii)** $y = Ae^{x^2/2}$
1.93 **(ii)** $z = Ae^{-5t} + 8/5$
1.95 **(ii)** $x = \pm\sqrt{Ae^t - 1}$

1.97 **(ii)** $r = \left[\ln\left(\frac{1}{4}\sin(2\theta) + C\right)\right]^2$

1.99 **(ii)** $s = (At^{3t}/e^{3t} - 3)^{1/3}$

1.101 **(ii)** $y = 3/(3\cos\theta - \cos^3\theta + C)$

1.103(a) For $R(0) = 10$, $R = -10e^{5t} + 20$. For $R(0) = 20$, $R = 20$. For $R(0) = 30$, $R = 10e^{5t} + 20$.

(d) $R = 20$; unstable

1.105(a) $dv/dt = -9.8 - 2v$

(b) $v = -4.9$

(c) $v(t) = C_1e^{-2t} - 4.9$

(d) **(ii)** -4.9

(e) $x(t) = -(C_1/2)e^{-2t} - 4.9t + C_2$

(f) **(ii)** $h(3) = 2988$ m

1.107(a) **(ii)** $a = (3\beta\sqrt{k}\,t/2 + C)^{2/3}$

(b) **(ii)** $a = \sqrt{2\beta\sqrt{k}\,t + C}$

(c) **(ii)** $a = Ae^{\beta\sqrt{\rho}\,t}$

1.109(d) **(ii)** $y(x) = Ce^{x^2/2+x}$

1.111 **(ii)** $y = Ae^{3x} + Be^{4x}$

1.113 **(ii)** $s = Ae^{-(5+\sqrt{41})t/2} + Bte^{-(5-\sqrt{41})t/2}$

1.115 **(ii)** $y(x) = A\sin\left(\sqrt{k^2+p^2}\,x\right) + B\cos\left(\sqrt{k^2+p^2}\,x\right)$

1.117 **(ii)** $y(x) = A\sin x + B\cos x + Ce^x + De^{-x}$

1.119 **(ii)** $u(x) = Ax^{-4} + Bx - 1$

1.121(e) $y = Ae^{3x} + Bxe^{3x} + 1/3$

1.123(d) $y = Ae^x + 2x + 2$

1.125(b) **(ii)** $y = (1/4)x^2$

1.127(a) $ap(p-1)(t-b)^{p-2} = -GM_E(t-b)^{-2p}/a^2$

(b) **(ii)** $r(t) = (9GM_E/2)^{1/3}(t-b)^{2/3}$

(c) Yes, and no

1.129(a) **(ii)** The equation is linear and homogeneous and all three solutions work.

(b) **(ii)** The equation is linear and inhomogeneous; y_2 and $y_1 + y_2$ work, but y_1 does not.

(c) **(ii)** The equation is non-linear. The solutions y_1 and y_2 work but their sum does not.

1.133(a) $y'y$

1.135 **(ii)** solution

1.137 **(ii)** solution

1.139 **(ii)** Not a solution

1.141(a) $dO/dt = k_1(O+E)$, $dE/dt = k_2(O+E)$

(b) $dO/dt = -k_1E$, $dE/dt = -k_2O$

(c) $dO/dt = k_1(O-E)$, $dE/dt = k_2(E-O)$

(d) $dO/dt = k_1E$, $dE/dt = -k_2O$

1.143(a) $dG_A/dt = r(1 - G_A/V_A)$, $dG_B/dt = r(G_A/V_A - G_B/V_B)$

1.145(a) Spring 2 pulls m_1 to the right. Springs 2 and 3 pull m_2 to the left.

(b) **(iv)** $x_1''(t) = -(k_1/m_1)x_1 + (k_2/m_1)(x_2 - x_1)$

(c) $x_2''(t) = (k_2/m_2)(x_1 - x_2) - (k_3/m_2)x_2$

1.147(a) $GM/(x^2+y^2)$

(b) **(ii)** $a_x = -GMx/(x^2+y^2)^{3/2}$, $a_y = -GMy/(x^2+y^2)^{3/2}$

(c) $d^2x/dt^2 = -GMx/(x^2+y^2)^{3/2}$, $d^2y/dt^2 = -GMy/(x^2+y^2)^{3/2}$

(d) **(ii)** Circle

1.149 **(ii)** $f(x) = Ae^{\sqrt{18}x} + Be^{\sqrt{18}x}$, $g(x) = A\sqrt{18}e^{\sqrt{18}x}/3 - B\sqrt{18}e^{\sqrt{18}x}/3$

1.151 **(ii)** $f(x) = Ae^{k_1x} + Be^{k_2x}$, $g(x) = (A/b)(k_1 - a)e^{k_1x} + (B/b)(k_2 - a)e^{k_2x}$
where $k_{1,2} = (1/2)\left(a + d \pm \sqrt{a^2 - 2ad + d^2 + 4bc}\right)$.

1.153(a) $T_A' = k_A(T_B - T_A)$, $T_B' = k_B(T_A - T_B)$

(d) **(ii)** $T_A = C_1 e^{-2(k_A+k_B)t} + C_2$, $T_B = -C_1\left(2k_B/k_A + 1\right)e^{-2(k_A+k_B)t} + C_2$

1.155(a) $T_A' = k_A(T_B - T_A)$, $T_B' = k_B(T_A + T_R - 2T_B)$

(c) **(ii)** $T_A = T_R + C_1 e^{-p_1 t} + C_2 e^{-p_2 t}$ where $p_{1,2} = (1/2)\left(k_A + 2k_B \pm \sqrt{k_A^2 + 4k_B^2}\right)$

1.157 **(ii)** $R\left(d^2R/dt^2\right) - (dR/dt)^2 - \left(\gamma R^2 - \delta R\right)(dR/dt) + \alpha\gamma R^3 - \alpha\delta R^2 = 0$

1.165 $x = 1.3$

1.169(c) **(ii)** $y = n\pi$ where n is any integer

1.173(a) $y(x) = C_1 J_0(k\rho) + C_2 Y_0(k\rho)$

(b) $y(x) = C_1 J_0(k\rho)$

(c) $k = 24.0\ \mathrm{m}^{-1}$, $k = 55.2\ \mathrm{m}^{-1}$, and $k = 86.6\ \mathrm{m}^{-1}$

(d) $f = 383$ Hz, $f = 879$ Hz, and $f = 1377$ Hz

1.175(a) $\psi(r) = Aj_l(r) + By_l(r)$ where j_l and y_l are "spherical Bessel functions" of the first and second kind respectively.

(b) $\psi(r) = Aj_l(r)$

(c) For $l = 0$, $r = 3.14$. For $l = 1$, $r = 4.49$. For $l = 2$, $r = 5.76$.

1.177(c) $y'(1/3) = 4/3$

(d) **(ii)** 16/9

(e) $y'(2/3) = 16/9$

(f) **(ii)** 64/27

(h) **(ii)** $y(1) = e$

1.179 (a) IV. (b) III. (c) V. (d) I. (e) II.

1.181 **(ii)** $r = (\sin\theta + 3\cos\theta)/10 + (7/10)e^{-3\theta}$

1.183 **(ii)** $y = 10^{x/2+1} - x$

1.185 **(ii)** $V = x^3 + (4/15)\left((\ln 2)^3 - 1\right)\left(e^{-2x} - e^{2x}\right)$

1.187 **(ii)** $y_{general}(t) = Ae^{kt}$, $y_{specific}(t) = pe^{kt}$

1.189 **(ii)** $u_{general}(x) = Ae^x - 3$, $u_{specific}(t) = 3e^x - 3$

1.191 **(ii)** $f_{general}(x) = Ae^{2x} + Be^{3x}$, $f_{specific}(x) = (e^{2x} - e^{3x})/(e^2 - e^3)$

1.193(a) $dP/dt = 120P - 12000$

(b) **(ii)** $P = 100$; unstable

(c) **(ii)** $P = 25e^{120t} + 100$

1.195(a) **(ii)** $y(x) = -\ln(C - e^x)$

(b) **(ii)** $y(x) = -\ln(e^2 - e^x + 1)$

1.201 $B(10) = 200e^{10} + 1000$

1.203(b) $dv/dt = -kv^2/m$

(c)

(d) The limit is $v = 0$ in both cases.

(e) **(ii)** $v(t) = 1/\left[(k/m)t + (1/v_0)\right]$

(f) **(ii)** $t_{1/10} = 9m/(kv_0)$

(g) **(ii)** $x(t) = (m/k)\ln(v_0 kt/m + 1)$

1.205(a) $m(t) = m_0 - kt$

(b) **(ii)** $d^2x/dt^2 = F/(m_0 - kt)$

(c) **(ii)** $x(t) = (F/k^2)(m_0 - kt)\ln(m_0 - kt) + C_1 t + C_2$

(d) **(ii)** $x(t) = \left(\dfrac{Fm_0}{k^2} - \dfrac{F}{k}t\right)\ln\left(\dfrac{m_0 - kt}{m_0}\right) + \dfrac{F}{k}t$

(e) **(ii)** $v(50\ \mathrm{s}) = 7.6 \times 10^4$ m/s, $x(50\ \mathrm{s}) = 1.6 \times 10^6$ m.

1.207 $P = C(T_0 - kz)^{mg/(Rk)}$

1.209(b) 0
(c) $M(t) = M_0 e^{-kt}$
(d) **(ii)** $t_{1/2} = (\ln 2)/k$
(e) **(ii)** 22 days
1.211(b) **(ii)** $ds/dt = -5$ and $dp/dt = 5$
(c) $dp/dt = k(f_0 - p)(s_0 - p)$
(d) **(ii)** $p(t) = f_0 - 1/(kt + C)$
(e) **(ii)** $p(t) = \dfrac{f_0 Ce^{k(s_0 - f_0)t} - s_0}{Ce^{k(s_0 - f_0)t} - 1}$
(f) **(ii)** f_0 for $s_0 > f_0$ and $s_0 = f_0$, s_0 for $s_0 < f_0$.
1.213(a) **(ii)** $P(t) = Ce^{2t}/(Ce^{2t} + 1)$
(b) $P(t) = e^{2t}/(7 + e^{2t})$
(c) 1

Chapter 2

2.1(a) 20.5
(b) 19.5
(c) 23
(d) None of the above
2.3(b) Yes
(c) Low
(d) $g(x)$
2.5 60.3°C
2.7(a) $f(25) = 5$
(b) 1/10
(c) **(i)** $1/10$
(ii) y decreases by $1/10$
(iii) $(1/10)\Delta x$
(iv) $\sqrt{23} \approx 4.8$
(d) $g(x) = 0.1x + 2.5$
(e) $f(23) \approx 4.8$
2.9 8
2.11 0.2
2.13 0.1
2.15 $3/e$
2.17(a) $22.5e^4 - 16e^4 = 6.5e^4 \approx 354.89$
2.19 $y = -0.75x + 6.25$
2.21(a) $\sin\theta \approx \theta$
(b) $d^2\theta/dt^2 + (g/L)\theta = 0.$
(c) $\theta(t) = C\cos\left(\sqrt{g/L}\,t\right) + D\sin\left(\sqrt{g/L}\,t\right)$
(e) Values near θ=0
2.23(a) Zero. ϕ will stay constant for a while.
(b) Decrease
(c) Increase
(e) $-\lambda v^2\phi + \lambda v^3$
(f) $\phi(t) = C\sin(\sqrt{\lambda}vt) + D\cos(\sqrt{\lambda}vt) + v$
(g) $t = 2\pi/\left(\sqrt{\lambda}v\right)$
2.25(a) $y = -e^{-t} + 1$
(c) $y = \ln 2$
(d) $-2y + 2\ln 2$
(e) $y = -\ln 2e^{-2t} + \ln 2$
2.27 $f(b) \approx f(a) + (b-a)f'(a)$
2.29(a) $4096a + 64b + c = 8$

(b) $128a + b = 1/16$

(c) $2a = -1/2048$

(d) $y = -(1/4096)x^2 + (3/32)x + 3$

(f) 0.071%

(g) 0.00274%

(h) 0.664%

2.33 $1 - x + x^2/2$

2.35 $1 + x - x^2/2$

2.37 $1 - x^2/2 - x^3$

2.39 $e + ex + ex^2$

2.41 $x + x^3/3 + 2x^5/15$

2.53 $\sum_{n=1}^{\infty}((-1)^{n-1}2^{2n-1}x^{2n})/(2n)!$

2.55(e) $-2(x - \pi/4) + (4/3)(x - \pi/4)^3$

(f) $\cos(2) = -0.416147$ and the fourth-order approximation is -0.416026, so they match to three decimal places.

2.59 $-2\sqrt{\pi}(x - \sqrt{\pi}) - (x - \sqrt{\pi})^2 + (4/3)\pi^{3/2}(x - \sqrt{\pi})^3$

2.61 $(5/2) + (3/2)(x - \ln 2) + (5/4)(x - \ln 2)^2 + (1/4)(x - \ln 2)^3$

2.63 $(x - \pi/2)^2$

2.65 $-1 - (1/2)(x - \pi)^2$

2.67 $(1/2\pi)(x - 2\pi) - (1/4\pi^2)(x - 2\pi)^2 + [(3 - 2\pi^2)/24\pi^3](x - 2\pi)^3$

2.83 $(1 + x)^2 = 1 + 2x + x^2$

2.95(a) $S_{22} = 1$

(b) This series diverges.

2.97(a) 10^{19}

(b) $\left(10^{20} - 1\right)/9$

(c) $\displaystyle\sum_{n=1}^{20} 10^{n-1}$

2.99(a) $1/5$

(b) $100 \times (1/5)^{99}$

(c) 125

(d) $\displaystyle\sum_{n=1}^{100} 100 \times (1/5)^{n-1}$

2.101(a) $S_{10} \approx 175.312$

2.103(a) $\displaystyle\sum_{n=1}^{\infty} 2(1.1)^{n-1}$

(b) 12.2102

(c) 114.54999

(d) $-20 + 20(1.1)^n$

(e) $+\infty$

2.107(a) No guarantee

(b) $\ln(1/4)$

(c) $-\ln(n + 1)$

(d) Diverges

2.111(a) $3/10, 3/100, 3/1000$

(b) $1/10$

(c) $1/3$

2.113 $0.\overline{9} = 1$

2.115(a) $2^{63} = 9,223,372,036,854,775,808$

(b) $2^{64} - 1 = 18,446,744,073,709,551,615$

2.117(a) 19

(b) $1,179,648,000$

(c) $2,359,291,500$

(d) $\approx 2.669 \times 10^{10}$

2.119(a) 112

(b) $\sum_{n=0}^{3} 2(2n)^2$

(c) 34%

(d) $\sum_{n=1}^{6} (1/2)(\ln((n+1)/2) = 2.183$

(e) $\sum_{n=1}^{50} 1.2^{n-1} \approx 45,497.191$

2.121(a) $\sum(1/3)(3/4)^n$

(b) $a_1 = 1/4$, and $r = 3/4$

(c) It converges to 1

2.123(a) $2 \times 3 \times 4 \times 5 \times 6 = 720$

(b) $\prod_{n=0}^{3} 2(3^n)$

(c) $\ln\left(\prod_{n=1}^{50} n\right)$

2.125(a) $\$24,290$

(b) $\$70,429$

2.127 1.0815

2.129 converges for $-1/2 < x < 1/2$

2.133 Convergent

2.135 Divergent

2.137 Inconclusive

2.139 Inconclusive

2.141 Converges for all x

2.143 Converges for $5/2 < x < 7/2$

2.145 Converges for all x.

2.167 diverges

2.169 converges

2.171 converges

2.173 converges

2.175 diverges

2.177 $-2 \leq x \leq 0$

2.179 $-1/e \le x < 1/e$

2.187(a) $\displaystyle\int_x^\infty e^{-t^3}dt = \left(\frac{1}{3x^2} - \frac{2}{9x^5}\right)e^{-x^3} + \frac{10}{9}\int_x^\infty \frac{1}{t^6}e^{-t^3}dt$

(b) approaching ∞

2.189(a) $\displaystyle\operatorname{erfc} x = \frac{e^{-x^2}}{\sqrt{\pi}\,x}\left(1 - \frac{1}{2x^2}\right) + \frac{3}{2\sqrt{\pi}}\int_x^\infty \frac{1}{t^4}e^{-t^2}dt$

(b) $\displaystyle a_{n+1} = \frac{c_n e^{-x^2}}{2x^{2n+1}},\ R_{n+1} = -\frac{c_n(2n+1)}{2}\int_x^\infty \frac{e^{-t^2}}{t^{2(n+1)}}dt$

(c) $|a_{n+2}/a_{n+1}| = (2n+1)/(2x^2)$

(d) greater when $n > x^2 - 1/2$

2.233 $f(0.1) = 2.1 \pm 0.016$

2.235(a) $\sin(1.5) \approx 0.9375$

(b) $\sin(1.5) \approx 0.997494$

2.237 $f(0.1) = 0.91 \pm .001$

2.239(a) $2x + 1$

(b) $x(t) = (1/2)(e^{2t} - 1)$

(c) no

2.245 $B(T) = b - A/RT,\ C(T) = b^2$

2.247(a) $c_0 = 0$

(b) $c_1 = 1$

(c) $2c_2 + (2\times 3)c_3x + (3\times 4)c_4x^2 + \ldots = -(c_0 + c_1x + c_2x^2 + c_3x^3 + \ldots)$

(d) $c_2 = 0$

(e) $c_3 = -1/(3!)$

(f) $y = x - x^3/(3!) + x^5/(5!) - x^7/(7!)$

(g) $y = \sin x$, yes it does

(h) $y(x) = 1 + x^3/6 + x^6/180$

Chapter 3

3.1 $Re = 3, Im = 0$

3.3 $Re = 0, Im = 0$

3.5 $Re = 15, Im = -3$

3.7 $Re = 1/2, Im = 3/4$

3.9 $Re = 49, Im = 0$

3.11 $Re = 0, Im = 3$

3.13 $Re = 39, Im = -80$

3.15 $Re = 5, Im = -31$

3.17 $Re = 15/169, Im = -36/169$

3.19 $Re = a^2 + b^2, Im = 0$

3.21 $x = \pm 10i$

3.23 $2 \pm i$

3.25

i^1	i^2	i^3	i^4	i^5	i^6	i^7	i^8	i^9	i^{10}	i^{11}	i^{12}
i	-1	$-i$	1	i	-1	$-i$	1	i	-1	$-i$	1

3.27 1

3.29 -1

3.31 $(-1 - i)z^{i-2}$

3.33 $-1/z^2$

3.35 $y(x) = Ae^{[-(1/5)+(2/5)i]x} + Be^{[-(1/5)-(2/5)i]x}$

3.37 $r = -2/3, s = 5$

3.39 $2 + 2i, -2 - 2i$

3.41 $2 + i, -2 - i$

3.45(a) $e^{ix} = \left(1 - x^2/2! + x^4/4! + \ldots\right) + \left(x - x^3/3! + x^5/5! + \ldots\right) i$

(c) $e^{x+iy} = \left(1 + x + \dfrac{x^2}{2} - \dfrac{y^2}{2} + \dfrac{x^3}{6} - \dfrac{xy^2}{2}\right) + i\left(y + xy + \dfrac{x^2y}{2} - \dfrac{y^3}{6}\right)$

3.47(a) $|3 - 3i| = 3\sqrt{2}, \phi_1 = 315°$

(b) $|-1 - 5i| = \sqrt{26}, \phi_2 = 259°$

(c) $|(3 - 3i)(-1 - 5i)| = 6\sqrt{13}, \phi_{product} = 574° = 214°$

(d) $-18 - 12i$

3.49(a) 13

(b) 169

3.51(a) $1/\sqrt{a^2 + b^2}$

(c) $1/k$

3.55(c) First quadrant for $p < 0$ and fourth quadrant for $p > 0$.

3.57(a) $3 + 4i$ is one possible answer

(b) No c exists

(c) No c exists

3.59 $\text{Re} = \sqrt{3}/2, \text{Im} = 1/2$

3.61 $\text{Re} = 1, \text{Im} = 0$

3.63 $\text{Re} = e^{-2}/2$, $\text{Im} = e^{-2}\sqrt{3}/2$

3.65(a) $7e^{0}$

(b) $7e^{i\pi}$

(c) $e^{i\pi/2}$

(d) $\sqrt{2}e^{i\pi/4}$

(e) $13e^{1.2i}$

(f) $13e^{-1.2i}$

3.71 A

3.73(a) $\rho = \sqrt{a^2 + b^2}$, $\phi = \tan^{-1}(b/a)$

3.75 $\cos(3x) = \cos^3 x - 3\cos x \sin^2 x$, $\sin(3x) = 3\cos^2 x \sin x - \sin^3 x$

3.77(b) $p = (3 \pm 6i)/5$

(c) $y_1 = e^{(3+6i)x/5}$, $y_2 = e^{(3-6i)x/5}$

(d) $y(x) = Ae^{(3+6i)x/5} + Be^{(3-6i)x/5}$

(e) $y(x) = e^{(3/5)x}\left(Ae^{(6i/5)x} + Be^{-(6i/5)x}\right)$

(f) $y(x) = e^{3x/5}\left[A\cos(6x/5) + iA\sin(6x/5) + B\cos(-6x/5) + iB\sin(-6x/5)\right]$

(g) $y(x) = e^{3x/5}((A + B)\cos(6x/5) + i(A - B)\sin(6x/5))$

(h) $y(x) = e^{3x/5}(A\cos(6x/5) + B\sin(6x/5))$

3.79 $x(t) = e^{-2t}(A\cos(2t) + B\sin(2t))$

3.81 $x(t) = e^{t/3}(A\cos(t/3) + B\sin(t/3))$

3.83(a) Doesn't oscillate; increases toward ∞

(b) Doesn't oscillate; decreases toward 0

(c) Oscillates with a period $2\pi/5$; neither

(d) Oscillates with a period $2\pi/5$; amplitude increases toward ∞

(e) Oscillates with a period $2\pi/5$; amplitude increases toward ∞

(f) Oscillates with a period $2\pi/5$; decreases to 0

3.85(a) $x(t) = e^{-(3/2)t}[A\cos(t/2) + B\sin(t/2)]$

(b) $x(t) = Ae^{-t} + Be^{-(5/2)t}$

(c) $\sqrt{40}$ kg/s

3.87(a) $8(d^2Q/dt^2) + R(dQ/dt) + Q = 0$

(b) $Q(t) = Ae^{(-R/16+\sqrt{R^2-32}/16)t} + Be^{(-R/16-\sqrt{R^2-32}/16)t}$

(c) $Q(t) = Ae^{(-1/4)t} + Be^{(-1/2)t}$

(d) $Q(t) = e^{(-1/4)t}(A\cos(t/4) + B\sin(t/4))$

(e) 0

3.89(f) $e^{-ix} = \cos x - i\sin x$

3.91(d) $x = A_2e^{3t} + B_2e^{-3t}$

3.93 $y(x) = -(1/2)\cos x$

3.95 $y(x) = -(1/2)\cos x$

3.97(a) $\mathbf{X}(0) = 3e^{i\pi/4}$, $x(0) = 3/\sqrt{2}$

(c) $\mathbf{X}(t) = 3e^{i\pi/4}e^{i(\pi/8)t}$

3.101(a) $|X| = |3 - 4i| = 5, \omega = 3$

(b) $\phi_{0x} = \tan^{-1}(-4/3) \approx -.93$

(d) behind by ϕ_{0x}

3.103(a) $\mathbf{X}''(t) + 2\mathbf{X}'(t) + 2\mathbf{X}(t) = 10e^{it}$

(b) $X_0 = 2 - 4i$

(c) $|X_0| = 2\sqrt{5}$

(d) 2π

(e) $x(0) = 2$

3.105 $(e^x/2)(\cos x + \sin x) + A$

3.107(d) $x(t) = (21/5)\cos(2t) - (1/5)\cos(3t)$

3.109(b) $f(x) = \pm(1/2)\cos(3x)$

3.111(a) $\mathbf{X_1}(t) = (13i)e^{(2\pi/3)it}$

(b) $\mathbf{X_2}(t) = (5 + 12i)e^{60\pi it}$

(c) $\mathbf{X_3}(t) = (5 - 12i)e^{-it^2}$

3.113(a) $Re(\mathbf{X}(t)) = a\cos(\omega t) - b\sin(\omega t)$, $Im(\mathbf{X}(t)) = a\sin(\omega t) + b\cos(\omega t)$

3.115(c) $Re\left[e^{i(\omega t+\delta+\phi_0)}\left(1 - e^{iN\delta}\right)/(1 - e^{i\delta})\right]$

3.117(a) $|z| = 1, \phi = \pi/8$

3.118(a) $\sqrt{y^2 + [x + (n-1)\delta]^2}$

(b) $\sqrt{y^2 + x^2} + \dfrac{x(n-1)}{\sqrt{y^2 + x^2}}\delta$

(c) $\dfrac{2\pi}{\lambda}\left(\sqrt{x^2 + y^2} + \dfrac{x(n-1)}{\sqrt{x^2 + y^2}}\delta\right)$

(d) $\displaystyle\sum_{n=1}^{N} A\cos\left[\omega t + \frac{2\pi}{\lambda}\left(\sqrt{x^2 + y^2} + \frac{x(n-1)}{\sqrt{x^2 + y^2}}\delta\right)\right]$

(e) $c = \dfrac{2\pi}{\lambda}\left(\sqrt{x^2 + y^2} - \dfrac{x\delta}{\sqrt{x^2 + y^2}}\right)$ and $d = \dfrac{2\pi x\delta}{\lambda\sqrt{x^2 + y^2}}$

(f) $Ae^{i\omega t}e^{i(c+d)}\left(1 - e^{idN}\right)/\left(1 - e^{id}\right)$

3.119(a) $Z = 2000 + 5500i$

(b) $\mathbf{I} = (1.8 \times 10^{-4} - 4.8 \times 10^{-4}i)e^{500it}$

(c) 5.1×10^{-4}

(d) $1.2\text{rad} \approx 70°$

(f) The amplitude of the current and the phase lag would remain the same. The plot of current as a function of time would be identical except shifted in time.

3.121 $Z = 1000 - 333i, I_0 \approx .474, \phi \approx -18.435°$

3.123(a) $-89.8°$

(b) approaches 0

(c) $R_2 = 66114\Omega$

3.125(a) **(i)** 0

(ii) Always ≥ 0.

(b) The power is never positive.

(c) The power will be positive half of the time.

3.127(a) $R - i/(\omega C)$

(b) $\mathbf{I_1} = (V_1 e^{i\omega_1 t})/(R - i/(\omega_1 C))$

(c) $\mathbf{I_2} = (V_2 e^{i\omega_2 t})/(R - i/(\omega_2 C))$

3.129 $Re(Z) = (5p-1)/26$, $Im(Z) = 5(1-5p)/26$, $|Z| = |5p-1|/\sqrt{26}$, $\phi_Z = -\tan^{-1} 5$

3.131 $Re(Z) = -7.36 \times 10^8$, $Im(Z) = 8.22 \times 10^8$, $|Z| = 1.10 \times 10^9$, $\phi_Z = 2.30$

3.133 $Re(Z) = 3/2$, $Im(Z) = -3\sqrt{3}/2$, $|Z| = 3$, $\phi_Z = -\pi/3$

3.135 $Re(Z) = \cos(p/2)$, $Im(Z) = \sin(p/2)$, $|Z| = 1$, $\phi_Z = p/2$

3.137 $Re(Z) = (3/2)\ln(2)$, $Im(Z) = -\pi/4$, $|Z| = \sqrt{(9/4)(\ln 2)^2 + \pi^2/16}$,
$\phi_Z = -\tan^{-1}\left[\pi/(6\ln 2)\right]$

3.139(a) $\sin x = i\left(e^{-ix} - e^{ix}\right)/2$

(b) $\cos x = \left(e^{ix} + e^{-ix}\right)/2$

(d) $\sinh(ix) = i\sin x$, $\cosh(ix) = \cos x$

(e) $\sin(ix) = i\sinh x$, $\cos(ix) = \cosh x$

3.141 $r = 0$, $s = \pm 3$

3.143(a) a circle

(b) a ray (half a line) starting at the origin

3.149(a) $2\pi/k$

(b) $2\pi/\omega$

(c) $\pi/600$ s

3.151 $y(x) = e^{-(2/5)x}(A\cos(6x/5) + B\sin(6x/5))$

3.153 $x(t) = e^{-(5/4)t}(A\cos(\sqrt{3}t/4) + B\sin(\sqrt{3}t/4))$

3.155 $(i/2)\left(e^{2x}/2 - x + C\right)$

3.157(a) $x(t) = Ae^{\left(-a+\sqrt{a^2-8}\right)t/2} + Be^{\left(-a-\sqrt{a^2-8}\right)t/2}$

(b) $-2\sqrt{2} < a < 2\sqrt{2}$

(c) Approach 0 ($a > 2\sqrt{2}$) or $\pm\infty$ ($a < 2\sqrt{2}$)

3.159 $x(t) = (3/17)\sin(2t) - (12/17)\cos(2t) + e^{-t}[A\cos(2t) + B\sin(2t)]$

Chapter 4

4.1(a) positive

(b) positive

4.7 $\partial f/\partial x = 1/y$, $\partial f/\partial y = -x/y^2$, $\partial^2 f/\partial x \partial y = -1/y^2$

4.9 $\partial f/\partial x = \cos x/\cos y$, $\partial f/\partial y = (\sin x \sin y)/(\cos^2 y)$, $\partial^2 f/\partial x \partial y = (\cos x \sin y)/(\cos^2 y)$

4.11 $\sin x/\cos^2 y$

4.13(a) Uphill

(b) Downhill

(c) Downhill

(d) Uphill

4.15(a) $1/R$

(b) $-V/R^2$

(c) $-1/R^2$

4.17(a) Negative

(b) Positive

(c) At $(0, 1)$, $\partial\phi/\partial x < 0$ and $\partial\phi/\partial y = 0$
At $(-1, 1)$, $\partial\phi/\partial x < 0$ and $\partial\phi/\partial y < 0$
At $(-1, -1)$, $\partial\phi/\partial x > 0$ and $\partial\phi/\partial y < 0$

(d) $\partial\phi/\partial x(1,, 1) = -1/2$, $\partial\phi/\partial y(1, 1) = 1/2$

(e) 1

4.19(a) $$\frac{2h^2\nu^4 e^{\frac{h\nu}{kT}}}{kT^2c^2\left(e^{\frac{h\nu}{kT}} - 1\right)^2}$$

(b) $$\frac{\partial I}{\partial \nu} = \left(\frac{2h\nu^2}{kTc^2}\right)\frac{(3kT - h\nu)\,e^{\frac{h\nu}{kT}} - 3kT}{\left(e^{\frac{h\nu}{kT}} - 1\right)^2}$$

4.21(a) $5/2$

(b) $\left(z_{42} - z_{22}\right)/2$

4.23 (b) and (c) only

4.25 (a), (b) and (d)

4.27(a) $V = (Swl - w^2l^2)/(2w + 2l)$

(b) $$\left(\frac{\partial V}{\partial w}\right)_l = \frac{l^2(S - w^2 - 2wl)}{2(w + l)^2}$$

(c) $$\left(\frac{\partial V}{\partial w}\right)_h = \frac{h^2(2S - 8wh - 2w^2)}{(w + 2h)^2}$$

(d) $(\partial V/\partial w)_l = 1/4\ \mathrm{m}^2$, $(\partial V/\partial w)_h = 0$

4.31(b) $dz/dt = 4(4t + 5)^{e-1}e^{3-t} - (4t + 5)^e e^{2-t}$

4.33(b)

$$\frac{dB}{dt} = \frac{\partial B}{\partial \rho_b}\left(\frac{\partial \rho_b}{\partial h}\frac{dh}{dt} + \frac{\partial \rho_b}{\partial T}\frac{dT}{dt}\right) + \frac{\partial B}{\partial \rho_{atm}}\frac{d\rho_{atm}}{dh}\frac{dh}{dt}$$

4.35(b) $ds/dt = 3(dp/dt) + 2(dg/dt) + (df/dt)$

(c) $29/15$ points per minute

4.37 $dP/dt \approx 10^{13}$ J/s

4.39 $-0.027c$ /s

4.41 $\dfrac{dq}{dt} = \dfrac{2q}{r}\dfrac{dr}{dt} - \dfrac{r^2}{kQ}\dfrac{dF}{dt}$

4.43 $dF = m\,da + a\,dm$

4.51(a) $2xy\dfrac{dx}{dt} + x^2\dfrac{dy}{dt} - e^x\dfrac{dx}{dt} + 2zt\dfrac{dz}{dt} + z^2 = 0$

(b) $dx/dt = (e^x - 2xy)^{-1}[x^2(dy/dt) + 2zt(dz/dt) + z^2]$

(c) 0.7

4.53 $df/dt = [h^2((dh/dt) + f\sin(gh)(g(dh/dt) + h(dg/dt))) + fe^{f/h}(dh/dt)](h^2\cos(gh) + he^{f/h})^{-1}$

4.55 $df/dt = \left[f - af^2(dg/dt) - bf^2(dh/dt)\right] / \left(cf^2 + t\right)$

4.57(b) $dy/dx = -(2x^3 + x)/(2y^3 + y)$

(c) $dy/dx = -0.13$

(d) $dy/dx = 0.13$

4.59(c) $d^2y/dx^2 = -25/y^3$

4.61(a) $m(x, y) = (e^x + 1)/(3y^2 + 1)$

4.63 $dy/dx = (y - 1)/(1 - x)$, $d^2y/dx^2 = 2(y - 1)/(1 - x)^2$

4.65(a) $x = e^y$

(b) $1 = e^y(dy/dx)$

(c) $dy/dx = 1/e^y$

(d) $dy/dx = 1/x$

4.67(a) $x = \sin y$

(b) $1 = \cos y(dy/dx)$

(c) $dy/dx = 1/\cos y$

(d) $dy/dx = 1/\sqrt{1 - \sin^2 y}$

(e) $dy/dx = 1/\sqrt{1 - x^2}$

4.69 $dy/dx = 1/(x^2 + 1)$

4.71(a) $D^2 = h^2 + s^2 - 2sh\cos(130°)$

(b) $dD/dt = (1/D)\,(s - h\cos 130°)\,ds/dt$

(c) $dD/dt = 569.5$ km/h

4.73(a) $di/dt = -50/9$ cm/s

4.75(a) $\dfrac{dK}{dt} = \dfrac{dY/dt - A\alpha L^{\alpha-1}K^{\beta}(dL/dt)}{A\beta L^{\alpha}K^{\beta-1}}$

(b) $\dfrac{dK}{dt} = \dfrac{dY/dt - A\alpha L^{\alpha-1}K^{\beta}(dL/dt) - AL^{\alpha}K^{\beta}(\ln L)(d\alpha/dt)}{A\beta L^{\alpha}K^{\beta-1}}$

4.77(a) $D_{\vec{u}}f(\pi, 1, 1) = -1$

(b) $D_{\vec{u}}f(\pi, 1, 1) = 0$

(c) $D_{\vec{u}}f(\pi, 1, 1) = 1$

(d) $D_{\vec{u}}f(\pi, 1, 1) = 1/\sqrt{2}$

4.79(a) $D_{\hat{u}}f(3,0,-1) = \sqrt{115}$

(b) $D_{\hat{u}}f(3,0,-1) = 103\sqrt{93}/93$

(c) $D_{\hat{u}}f(3,0,-1) = 127\sqrt{141}/141$

4.81(a) $df/dt = (\partial f/\partial x)(dx/dt) + (\partial f/\partial y)(dy/dt) + (\partial f/\partial z)(dz/dt)$

4.83(a) $\hat{u} = \left(0.3\hat{\imath} + 0.1\hat{\jmath} + 0.2\hat{k}\right)/\sqrt{0.14}$, $D_{\hat{u}}f = -1/\sqrt{0.14}$

4.85 $y = x, +y, +x$

4.87(a) $\hat{u} = (\cos\phi)\hat{\imath} + (\sin\phi)\hat{\jmath}$

(b) $D_{\hat{u}}\rho = e^{-x^2+y^2}\left(\sin(\pi y^2/2)\cos\phi\left[(\pi/2)\cos(\pi x/2) - 2x\sin(\pi x/2)\right]\right.$
$\left. + y\sin(\pi x/2)\sin\phi\left[\pi\cos(\pi y^2/2) + 2\sin(\pi y^2/2)\right]\right)$

(c) $D_{\hat{u}}\rho(1,1) = e^{-3}(-2\cos\phi + 2\sin\phi)$

(d) At $\phi = 3\pi/4$ we get $D_{\hat{u}}\rho \approx 0.141$.

4.95(a) $\vec{\nabla}f(0,0) = \vec{0}$, $\vec{\nabla}f(0,1) = 2\hat{\jmath}$, $\vec{\nabla}f(0,-1) = -2\hat{\jmath}$, $\vec{\nabla}f(1,0) = 2\hat{\imath}$, $\vec{\nabla}f(-1,0) = -2\hat{\imath}$

4.97(a) $2\sqrt{13}$

(b) $-2\sqrt{13}$

(c) $-4\hat{\imath} - 6\hat{\jmath}$

(d) $\hat{u} = (3/\sqrt{13})\hat{\imath} - (2/\sqrt{13})\hat{\jmath}$

(e) one other

4.99(a) $-\left(x/\sqrt{25-x^2-y^2}\right)\hat{\imath} - \left(y/\sqrt{25-x^2-y^2}\right)\hat{\jmath}$

(b) $2x\,\hat{\imath} + 2y\,\hat{\jmath} + 2z\,\hat{k}$

4.101(a) $t = 3\pi/4$

(b) $t = 3\pi/2$

4.105 $-2\hat{\imath} - 4\hat{\jmath} + 6\hat{k}$

4.107(a)
$$\vec{E} = \left(\frac{k(x-x_0)}{\left[(x-x_0)^2+y^2\right]^{3/2}} - \frac{k(x+x_0)}{\left[(x+x_0)^2+y^2\right]^{3/2}}\right)\hat{\imath}$$
$$+\left(\frac{ky}{\left[(x-x_0)^2+y^2\right]^{3/2}} - \frac{ky}{\left[(x+x_0)^2+y^2\right]^{3/2}}\right)\hat{\jmath}$$

(b) In the positive x-direction

(c) 0.6 radians

4.109(a) $\partial f/\partial x = 14$, $\partial f/\partial y = -8$

(b) $14x - 8y$

(c) $\vec{u} = -14\hat{\imath} + 8\hat{\jmath}$

(d) $\partial f/\partial x = (\alpha d - \beta b)/(ad - bc)$, $\partial f/\partial y = (\alpha c - \beta a)/(ad - bc)$

(e) $ab - cd = 0$ means parallel vectors

4.111(a) $g(x,y) = 7 + (1/14)(x-40) + (3/7)(y-3)$

(c) $f(41, 2.9) = 7.02922$, $g(41, 2.9) = 7.02857$

(d) $f(50,5) = 8.66$, $g(50,5) = 8.57$

4.113(a) $\left(\dfrac{\partial^{11} f}{\partial x^7 \partial y^4}(0,0)\right)\dfrac{1}{7!\times 4!}x^7y^4$

(b) $\left(\dfrac{\partial^{11} f}{\partial x^7 \partial y^4}(-3,\pi)\right)\dfrac{1}{7!\times 4!}(x+3)^7(y-\pi)^4$

4.115 $f(1/2,1/2)\approx -0.613$

4.117 $z(x,y)\approx 3+(1/2)(x-6)-(3/2)(y-2)$

4.119 $\sin(x+y^2)\approx x+y^2-x^3/6-x^2y^2/2$

4.121(a) $e^{x+2y}=1+x+2y+(1/2)x^2+2xy+2y^2+(1/6)x^3+x^2y+2xy^2+(4/3)y^3+\ldots$

(b) $e^{x+2y}=1+x+2y+(1/2)x^2+2xy+2y^2+\ldots$

(c) $2e^{x+2y}=2+2x+4y+x^2+4xy+4y^2+\ldots$

4.123(a) $f(x,y,z)=\displaystyle\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{p=0}^{\infty}\left(\frac{\partial^{n+m+p} f}{\partial x^n \partial y^m \partial z^p}(x_0,y_0,z_0)\right)\frac{1}{n!m!p!}(x-x_0)^n(y-y_0)^m(z-z_0)^p$

(b) $f(x,y,z)=y, f(x,y,z)=y+x^2+(5/2)yz$

(c) $f(0.01,0.02,-0.01)=0.0191245$, first-order approximation 0.02, second-order approximation 0.0196

4.125(a) $v_{AC}\approx v_{BC}+v_{AB}$

(b) $v_{AC}\approx c-(v_{AB}-c)(v_{BC}-c)/(2c)$

4.127(a) $f(\theta_1,\theta_2,\dot\theta_2,\ddot\theta_1,\ddot\theta_2)\approx 2(g/L)\theta_1+2\ddot\theta_1+\ddot\theta_2$

(b) $2\ddot\theta_1+\ddot\theta_2+(2g/L)\theta_1=0, \ddot\theta_1+\ddot\theta_2+(g/L)\theta_2=0$

(c) $A=-\left(1/\sqrt{2}\right)B$

(e) $5\sqrt{2}\,^\circ$

4.129 $(-3,-4)$ is a local maximum, $(5/2,4)$ is a local minimum, $(5/2,-4)$ and $(-3,4)$ are saddle points

4.133(a) saddle point.

4.135 Absolute maximum: $2\pi-2$. Absolute minimum: $2-6\pi$.

4.137 Absolute maximum: 48. Absolute minimum: $-16/729$.

4.139 $64C_0D^2/49$

4.141 $(2/3,5/3,-1/3)$

4.145 $\left(1/2,1/\sqrt{8},1/2\right)$

4.147 $(13/9,15/9)$

4.149(a) $(x_3=4/5,y_3=3/5,x_4=-3/5,y_4=4/5)$, and $(x_3=-4/5,y_3=3/5,x_4=3/5,y_4=4/5)$.

4.151(a) $y=(y_1-y_0)x+y_0$

(b) $y=(1/2)(y_2-y_0)x+(1/6)(5y_0+2y_1-y_2)$

4.155(a) $\vec\nabla f=(2x+4y+2)\hat i+(6y+4x)\hat j$

(b) $g(x,y)=x-y=13$ so $\vec\nabla g=\hat i-\hat j$

(c) $2x+4y+2=\lambda, 6y+4x=-\lambda$

(d) $(8,-5)$

4.157 $W=L=260/7, H=390/7$

4.159 $(2/3,5/3,-1/3)$

4.161 $(0.393,0.786,0.772)$

4.163 $(1/2, 1/\sqrt{8}, 1/2)$

4.167 $(6, 0), (3/2, \pm\sqrt{135}/2)$

4.169 $(0, 0, 0, 0)$ and $(2/9, 4/27, -10/27, -26/81)$

4.173(a) $\sqrt{(x-50)^2+(y-10)^2+(z-20)^2}+\sqrt{(x-30)^2+(y-20)^2+(z-30)^2}$

(b) $x^2+y^2+z^2=100$

4.175(a) $v_x = v_y = \sqrt{E/m}$

(b) $\lambda = 2/(mg)$

(c) $2(dE)/(mg)$

4.177(a) $L = \left(\dfrac{s+c}{T(\alpha+\beta)}\right)^{1/(\alpha+\beta-1)}$

(b) $\lambda = (c\alpha - s\beta)/(\alpha+\beta)$

(c) $w(s\beta - c\alpha)/(\alpha+\beta)$

(d) \$1200

4.179(a) $-P$

4.181(a) 0 (not changing)

(b) $dS/dt = (nR/V)(dV/dt)$

4.183(a) $dT/dt = (P/nR)(dV/dt)$

(b) $dU/dt = (3P/2)(dV/dt)$

(c) $dS/dt = (5P/2T)(dV/dt)$

(d) entering

4.185(a) Six

(b) 8715 J

4.187(a) $C_V = (f/2)nR$

(b) $C_P = \dfrac{f}{2}nR + \dfrac{nRV\left(an^2+PV^2\right)}{PV^3 - an^2 + 2abn^3}$

4.193(a) $3nR/(2T)$

(b) $5nR/(2T)$

(d) First expression: $\dfrac{\partial S}{\partial T} = \dfrac{n^2R^2}{PV+nRT} + \dfrac{3nR}{2T}$

Second expression: $\dfrac{\partial S}{\partial T} = \dfrac{2n^2R^2}{PV+2nRT} + \dfrac{3nR}{2T}$

4.195(d) $(\partial f/\partial x)_y = 0$

(e) $(\partial f/\partial x)_\rho = -x/y$

(f) $(\partial f/\partial x)_\phi = y/x$

4.197(a) **(i)** $\partial a_1/\partial x = 1+y^2$ and $\partial a_1/\partial y = 2xy$.

(ii) $da_1 = \left(1+y^2\right)dx + 2xydy$.

(iii) $da_1 = \left(1+5x^4\right)dx$.

(b) **(i)** $\partial a_2/\partial x = 3x^2y$ and $\partial a_2/\partial y = (1/2)y^{-1/2}+x^3$.

(ii) $da_2 = 3x^2ydx + \left((1/2)y^{-1/2}+x^3\right)dy$.

(iii) $da_2 = \left(5x^4+1\right)dx$.

4.199(a) $dH = T\,dS + V\,dP$

4.201 $(\partial T/\partial P)_S = (\partial V/\partial S)_P$

4.203 $\partial f/\partial x = 2xy^3$, $\partial^2 f/\partial x\partial y = 6xy^2$, $\vec{\nabla} f = 2xy^3\hat{\imath} + 3x^2y^2\hat{\jmath}$, $D_{\vec{u}}f = 3x^2y^2$

4.205 $\partial f/\partial x = y/(x+y)^2$, $\partial^2 f/\partial x\partial y = (x-y)/(x+y)^3$, $\vec{\nabla} f = \left[y/(x+y)^2\right]\hat{\imath} - \left[x/(x+y)^2\right]\hat{\jmath}$, $D_{\vec{u}}f = \left[y\cos(70°) - x\sin(70°)\right]/(x+y)^2$

4.207 (a), (b)

4.209(a) $\partial k/\partial T = AE_a e^{-E_a/RT}/(RT^2)$

(b) $\partial k/\partial E_a = -Ae^{-E_a/RT}/RT$

4.211(b) $dI/dt = (\partial I/\partial V)(dV/dt) + (\partial I/\partial R)(dR/dT)(dT/dt) + (\partial I/\partial L)(dL/dt)$

4.213(a) $dc/dt = \partial c/\partial t + (\partial c/\partial x)(dx/dt)$

4.215(a) $\vec{\nabla} z = -8\hat{\imath} - 8\hat{\jmath}$

(d) $\vec{\nabla} f = 8\hat{\imath} + 8\hat{\jmath} + \hat{k}$

4.219(a) $f(x,y) \approx x, f(0.01, 0.01) \approx 0.01$

(b) $f(x,y) \approx x - 2x^2 - xy, f(0.01, 0.01) \approx 0.0097$

(c) $f(0.01, 0.01) = 0.00971$

(d) $f(x,y) \approx (1/4) + (1/8)(x-1) - (1/16)(y-1), f(1.01, 0.9) \approx 0.2575$

(e) $f(x,y) \approx (1/4) + (1/8)(x-1) - (1/16)(y-1) - (1/16)(x-1)^2 + (1/64)(y-1)^2$, $f(1.01, 0.9) \approx 0.25766$

(f) $f(1.01, 0.9) \approx 0.257653$.

4.221(a) $R_1 + R_2 + R_3$

(b) $50R_1 + 20R_2 + 10R_3 \le 1000$, $2R_1 + 5R_2 + 10R_3 \le 200$, $R_1 \ge 0$, $R_2 \ge 0$, $R_3 \ge 0$

(d) 42.9 L

(e) 42.9 L

(f) 42.9 L

4.223(a) $\partial W/\partial v = -5.72v^{-0.84} + 0.0684Tv^{-0.84}$

4.225(a) $U \approx GM\left[-\frac{1}{R} - \frac{J_2}{2R} + \left(\frac{2+3J_2}{2R^2}\right)(r-R) - \left(\frac{1+3J_2}{R^3}\right)(r-R)^2 + \frac{3J_2}{2R}l^2\right]$

(b) $U \approx GM\left[-\frac{1}{R} - \frac{J_2}{2R} + \left(\frac{2+3J_2}{2R^2}\right)(r-R) - \left(\frac{1+3J_2}{R^3}\right)(r-R)^2 + \frac{3J_2}{2R^3}y^2\right]$

(c) $\vec{F} \approx -\frac{GM}{R^2}\left(3J_2\frac{y}{R}\hat{y} + \left[1 + \frac{3}{2}J_2 - 2\left(1+3J_2\right)\frac{r-R}{R}\right]\hat{r}\right)$

Chapter 5

5.1(a) 20

(c) $dm = 3x\,dx$

(d) $\int_1^3 3x\,dx$

(e) 12

5.3(a) $2/n$

(b) $1 + 2(i-1)/n$

(c) $3 + 6(i-1)/n$

(d) $6/n + 12(i-1)/n^2$

(e) $12 - 6/n$

(f) 12

5.5(a) $2/3$ m

(b) $4/3$ m

5.7 $M = 2cL/\pi$, $x_{COM} = L/2$, $I = (cL^3/\pi^3)(\pi^2 - 4)$

5.9 $M = (a/b)\left(1 - e^{-bL}\right)$

$x_{COM} = (1/b)\left(1 - (1 + bL)e^{-bL}\right) / \left(1 - e^{-bL}\right)$

$I = (a/b^3)\left(2 - \left(2 + 2bL + b^2L^2\right)e^{-bL}\right)$

5.11 $V = kcL^2/2$

5.15(a) 20

(b) $m(10)\,dx$

(c) $\int_{10}^{13} m(x)\,dx$

5.17(a) 6 inches

(b) $g(t)dt$

(c) $\int_7^{10} g(t)dt$

5.19(a) 24 Coulombs

(b) $I(t)\,dt$

(c) $1/10\pi$ Coulombs

(d) 0 Coulombs

5.21 $(1/6)\rho g W H^3$

5.23(a) mgh

(b) $kL^2/2$

(c) $Gm_A m_E/R$

5.25(c) $2\pi\rho a/\left(\rho^2 + b^2\right)d\rho$

(d) $\pi a \ln\left(\left(R^2/b^2\right) + 1\right)$

(e) a: mass; b: distance

5.29(a) 18

(b) 36

5.31(d) $y_{com} = 12/5$

5.33 $x_{COM} = 3L/4$, $y_{COM} = (3/10)cL^2$

5.35(a) $ka^3/2$

(b) $x_{COM} = 2a/3, y_{COM} = a/2$

(c) $I = ka^5/4$

(d) $I = k\left(a^5/4 - 2ba^4/3 + b^2a^3/2\right)$

5.37 $4(e-1)\pi\rho_0 R^3/(3e)$

5.41 $V = \pi H^3$

5.43(a) $\rho(h) = 1.2e^{-1.16\times10^{-4}h}$

(b) 10344.8 kg

5.45(a) rectangular solid with square base

(b) $dV = w^2\,dy$

(c) $w = L(H-y)/H$

(d) $V = L^2H/3$

5.47 $MR^2/2$

5.51(b) $\sqrt{\rho^2+h^2}$

(c) $dV = \dfrac{2kQ\rho\,d\rho}{R^2\sqrt{\rho^2+h^2}}$

(d) $(2kQ/R^2)\left(\sqrt{R^2+h^2}-h\right)$

(e) kQ/h

5.53 $3R/8$

5.55 horizontal slices, $1/12$

5.57 horizontal slices, $\left(\alpha/k^2\right)\left(e^{kH}-1\right)\left(e^{kW}-1\right)$

5.59 vertical slices, 0

5.63(a) $4/\pi\,dx$

(b) $2\sqrt{2}/\pi\,dy$

(c) $-16/\pi^2$

5.65(a) $\int_0^{10}\int_0^7 kye^{xy}\,dx\,dy = 3.59\times10^{29}\,k$

(b) $k\int_0^7\int_0^{10} ye^{xy}\,dy\,dx = 3.59\times10^{29}\,k$

5.67 kW^2H

5.69(a) 0

(b) Half the surface is above the xy-plane and half is below.

5.71(a) $250(e^3-e^{-3})/3$

(b) $250(e^3-e^{-3})/3$

(c) $250(2e^3-2)/3$

5.75(a) $(k/\beta\alpha)\left[\cos(\alpha L)-1\right]\left[e^{-\beta H}-1\right]$

(b) 6000 g

5.77 97.2

5.79(a) $g(y)\,(F(b)-F(a))$

(b) $(G(d)-G(c))\,(F(b)-F(a))$

(c) $(G(d)-G(c))\,(F(b)-F(a))$

5.81 $R^4/8$

5.83 329

5.85 1/2

5.87 $\int_4^6 \int_3^7 f(x,y)\,dy\,dx$

5.89 $\int_0^{16} \int_0^{x/2} f(x,y)\,dy\,dx$

5.91 $\int_0^1 \int_0^{\cos^{-1}(y)} f(x,y)\,dx\,dy$

5.93 $\int_0^5 \int_{\sqrt{25-y^2}}^{\sqrt{25-y}} f(x,y)\,dx\,dy$

5.95 $\int_0^3 \int_y^{(-y+9)/2} f(x,y)\,dx\,dy$

5.97(a) $\int_0^2 \int_0^{x^3}\,dx\,dy = 4$

(b) $\int_0^2 x^3\,dx = 4$

5.101(a) $V = (4kQ/\pi R^2) \int_0^R \int_0^{\sqrt{R^2-x^2}} 1/\sqrt{x^2+y^2}\,dy\,dx$

(b) $I = (4M/\pi R^2) \int_0^R \int_0^{\sqrt{R^2-x^2}} x^2+y^2\,dy\,dx$

5.105 $2cH^7/105$

5.107 $4R^3H/(3\pi)$

5.111(a) $\left(4cL^2/k\pi^2\right)\left(1-e^{-kL}\right)$

(b) $\left(\dfrac{\pi}{k}\right)\dfrac{1-e^{-kL}(kL+1)}{1-e^{-kL}}$

5.113 $M = 7500, x_{COM} = 3/2, y_{COM} = 0, z_{COM} = 125/7$

5.117(a) $-\sqrt{1-z^2} \le y \le \sqrt{1-z^2}, -1 \le z \le 1$

(b) $-\sin x \le y \le \sin x, 0 \le x \le \pi$

5.119 $(1/6)ML^2$

5.121 $kL^4/3$

5.123 $(2/5)ML^2$

5.129 0

5.131 0

5.135 $R^4/8$

5.137 The potential is $2kQ/R$ and the moment of inertia is $(1/2)MR^2$.

5.139 $ck \int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} x^2 / \left[\left(x^2+y^2\right)\sqrt{x^2+y^2+h^2}\right]\,dy\,dx$

5.141 $kR^4/2$

5.143(a) Arbitrarily choosing to place the flat edge on the x-axis, $\sigma = ky$

(b) $(2/3)kR^3$

(c) $\sigma = k\rho\sin\phi$

(d) $(2/3)kR^3$

5.147(c) ρ: 0 to $\sin(3\phi)$, ϕ: 0 to $\pi/3$

(d) $\pi/4$

5.149(b) $\rho = 3, \rho = 6\cos\phi$

(c) $\pm\pi/3$

(e) $(9\sigma/2)(\pi/3 - \sqrt{3}/2)$

(f) $3\pi\sigma$

(g) $(6\pi - 9\sqrt{3}/2)\sigma$

5.151 $(k/6)\left(2^{3/2}-1\right)$

5.155 1

5.157(b) 6

(d) $v = 1$

(e) $u = 1$

(f) $y = 0$ becomes $u = 0$ for $-1 \le x \le 0$ and $v = 0$ for $0 \le x \le 1$

(h) $4u^2 + 4v^2$

(i) 6

(j) 4

5.159 $(e-2)/3$

5.161 $(2/3)\tanh^{-1}(1/2)$

5.163(a) $(\rho, \phi, z) = (\sqrt{2}, -\pi/4, 4), (r, \theta, \phi) = (\sqrt{18}, 0.339, -\pi/4)$

(b) $(\rho, \phi, z) = (\sqrt{2}, 3\pi/4, 4), (r, \theta, \phi) = (\sqrt{18}, 0.339, 3\pi/4)$

(c) $(x, y, z) = (-2.5, 2.5 * \sqrt{3}, -1), (r, \theta, \phi) = (\sqrt{26}, 1.77, 2\pi/3)$

(d) $(x, y, z) = (0, 0, 1), (\rho, \phi, z) = (0, 0, 1)$

5.165(a) $r = 5/\cos\theta$

(b) Cartesian: $z = \sqrt{2}\sqrt{x^2+y^2}$, Cylindrical: $z = \sqrt{2}\rho$

(c) Cylindrical: $\rho = 2$, Spherical: $r = 2/\sin\theta$

5.167(a) $r(t) = 4000, \theta(t) = \pi/3, \phi(t) = \pi t/12$

(b) $x(t) = 2000\sqrt{3}\cos(\pi t/12), y(t) = 2000\sqrt{3}\sin(\pi t/12), z(t) = 2000$

5.169(a) c is 1/distance. k is mass/volume.

(d) $z = c\rho^2, D = k\rho/z.$

(e) $(k\rho/z)\rho\, d\rho\, d\phi\, dz$

(f) 0 to 2π

(h) $\int_0^H \int_0^{\sqrt{z/c}} d\rho\, dz$

(i) $(4\pi k/9)(H/c)^{3/2}$

(j) mass

5.171 $\left(\pi H^3/6\right)\left(e^{R^2}-1\right)$

5.173 $4\pi R^5/15$

5.175 $(\pi/10)\left(2H^3R^2 + HR^4\right)$

5.177 $4\pi R^5/5$

5.179 For the hemisphere, $I = (2/5)MR^2$ and $V = 3kQ/(2R)$. For the sphere, $I = (4/5)MR^2$ and $V = 3kQ/R$.

5.181(a) r

(b) $r\, d\theta$

(c) radius: $r\sin\theta$, arc length: $r\sin\theta\, d\phi$

(d) $r^2\sin\theta\, dr\, d\theta\, d\phi$

5.183 $\pi k R^3 H/6$

5.185(a) $R = \sqrt{3/2}\, H$

(b) $M = k\pi R^4(1-\sqrt{2/3})/2$

5.187(a) $(2M/5)\left(R_2^5 - R_1^5\right) / \left(R_2^3 - R_1^3\right)$

(b) $2MR_2^2/5$

(c) $(2/3)MR_1^2$

5.189 $\pi DHR^4/10$

5.191(a) $D = D_0 e^{k_1\rho} e^{-k_2 z}$

(b) D_0: density, k_1 and k_2: distance^{-1}

(c) $D_0 \int_0^H \int_0^R \int_0^{2\pi} e^{k_1\rho} e^{-k_2 z} \rho^3 \, d\phi \, d\rho \, dz$

(d) $\left(2\pi D_0/(k_2 k_1^4)\right)\left(1 - e^{-k_2 H}\right)\left(e^{k_1 R}\left[k_1^3 R^3 - 3k_1^2 R^2 + 6k_1 R - 6\right] + 6\right)$

5.193(c) $y^2 + z^2 = 1$

(d) z goes from $-\sqrt{1-\rho^2\sin^2\phi}$ to $\sqrt{1-\rho^2\sin^2\phi}$

(f) $I_z = \int_0^{2\pi}\int_0^1 \int_{-\sqrt{1-\rho^2\sin^2\phi}}^{\sqrt{1-\rho^2\sin^2\phi}} \rho^3 \, dz \, d\rho \, d\phi \Big/ \left[\int_0^{2\pi}\int_0^1 \int_{-\sqrt{1-\rho^2\sin^2\phi}}^{\sqrt{1-\rho^2\sin^2\phi}} \rho \, dz \, d\rho \, d\phi\right]$

5.195(a) 0 to 2π

(c) $r = 1/\sqrt{1+\cos^2\theta}$

(d) $\int_0^{\pi}\int_0^{1/\sqrt{1+\cos^2\theta}}\int_0^{2\pi} r^4 \sin^2\theta \, d\phi \, dr \, d\theta$

5.197(a) 70

(b) -70

(c) 0

(d) 28

5.199(a) 60π

(b) -60π

(c) 0

(d) 0

5.201 8

5.203 $77/10 + 16\sqrt{2}/5 - 1/\sqrt{2}$

5.205 0

5.209 $\ln 2$

5.211 $\left(\sqrt{14}/8\right)\sinh 2 \approx 1.7$

5.215 $t = 125\left[\left(1 + (36/25)k^2 x_f^{2/5}\right)^{3/2} - 1\right] / \left[108\sqrt{2gk^5}\right]$

5.217(b) -33

(c) 65

(d) -65

(e) $(13/5)(4\,\hat{\imath} - 3\,\hat{\jmath})$

5.219 $k/R_2 - k/R_1$

5.221 $k/(H - 4\pi p/\omega) - k/H$

5.223 0

5.225(a) $2\pi\rho|\vec{B}|$

(b) $|\vec{B}| = \mu_0 I/(2\pi\rho)$

5.229(b) $2|\vec{B}|L$

(c) $|\vec{B}| = \mu_0 J/2$

5.235(c) $\rho\, d\phi$

(d) $\sqrt{(\rho'(\phi))^2 + \rho^2}\, d\phi$

5.237(a) $x = 2\cos u + v^2, y = v + 3, z = \sin u$

(b) $x = 2\cos u - v^2, y = v, z = \sin u$

(c) $x = \cos u + v^2, y = v, z = (1/2)\sin u$

5.245(a) $x^2 + y^2 \leq 9, z = 5$

(b) $\rho \leq 3, z = 5$

(c) $x = v\cos u, y = v\sin u, z = 5, 0 \leq v \leq 3, 0 \leq u \leq 2\pi$

5.247(a) $x = 7\sin\theta\cos\phi, y = 7\sin\theta\sin\phi, z = 7\cos\theta, 0 \leq \phi \leq 2\pi, 0 \leq \theta \leq \pi$

(b) $x = 7\sin\theta\cos\phi + 3, y = 7\sin\theta\sin\phi + 7, z = 7\cos\theta - 4, 0 \leq \phi \leq 2\pi, 0 \leq \theta \leq \pi$

5.251(a) $x = x, y = \sin x\cos\theta, z = \sin x\sin\theta, 0 \leq \theta \leq 2\pi$

5.255(a) 60

(b) -60

(c) 60

5.257(a) 0

(b) 0

(c) $1000\pi/3$

(d) $500\pi/3$

(e) $500\pi/9$

5.259 $-44/15$

5.261 $\ln 2\,(1 - \cosh 2) + (\pi/9)\left(\cosh^3 2 - 1\right) \approx 16$

5.263(a) 1 kg

(b) < 1 kg

(c) > 1 kg

5.265(a) $4\pi/3$

5.267(a) 27π

(b) 108π

5.271(a) width $|\vec{a}|$, height h

(b) $|\vec{a}|\ |\vec{b}|\ \sin\theta$

5.273(a) Q/ε_0

(b) $4\pi r^2 \left|\vec{E}\right|$

(c) $\left|\vec{E}\right| = Q/\left(4\pi\varepsilon_0 r^2\right)$

5.275(a) $\left|\vec{E}\right| = Q/2\pi\varepsilon_0 r$

(b) $\propto r^{-9}$

5.277(b) $\sigma\pi R^2$

(d) $\left|\vec{E}\right| 2\pi r^2$

(e) 0

(f) σ/ε_0

5.279(a) $|B_0|\pi\omega R^2 \sin\theta$

5.281(a) $\mu_0 IL \ln 2/(2\pi)$

(b) $-\mu_0 I_f L \ln 2/(2\pi\tau)$

(c) 6.16×10^{-11} s

5.283 $(GmM)/(w(w+L))\hat{i}$

5.293(a) $-3GMm/R^2$

(b) $\left[R^2 + L^2 - 2Lz\right]^{1/2}$

(c) $3GMm/R^2$

(d) $-GMm/L^2$

(e) $-GMm/L^2$

5.295 $\vec{B}_z = \mu_0 IR^2/\left[2\left(H^2+R^2\right)^{3/2}\right]$

5.297(a) 150

(b) 375

(c) 225

(d) 1125/2

5.299 $kL^4/96$

5.301 $(1/2)\ln(5/3)$

5.303 0

5.305 $(1/160) + (1/4\pi)\cos(\pi/8) - (2/\pi)\sin(\pi/8)$

5.307 $\pi^2 R^4/4$

5.311(a) $(1/4)\left(4e + e^2 - 1\right)$

(b) 2π

5.313 $4\pi R^3/3$

5.315 $\pi\left(2R^2H + (4/3)RkH^3 + (2/5)k^2H^5\right)$

5.317 $\pi R^3/6$

5.319 $M = a\pi^2R^5/5$, $Q = \pi bR^2(\ln(16) - 2)$, center of mass at origin, $I_z = 3a\pi^2R^7/28$, $V = 4\pi kbR(1 - \ln 2)$

5.321 $x_{com} = y_{com} = 0$, $z_{com} = H/4$, $I_z = (2/5)ML^2$, $V = (4/3)k\beta HL^2$

5.323 $4W^2H/5$

5.327(b) $(4, -2)$ and $(4, 2)$

(d) 464/15

5.329 $2kR/3$

5.331(b) mgH

5.333(b) $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2}\, dx\, dy$

(c) $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dx\, dy$

(e) $\displaystyle\int_{0}^{\infty}\int_{0}^{2\pi} e^{-\rho^2}\, \rho\, d\phi\, d\rho$

(f) $S = \sqrt{\pi}$

(h) $\sqrt{\pi}/2$

Chapter 6

6.1(c) $x_1 = 0, x_2 = 1$

6.3(a) yes

(b) no

(c) yes

(d) no

6.7(a) $x_1 = -1, x_2 = 1$

(b) $a = b = 1/2$

6.9(a) yes, yes

(b) no, yes

(c) no, no

6.17(a) $1/2$

(d) $-1/4$

(e) yes

(g) $x_1(t) = 2\cos t - 4\cos(\sqrt{7}t), x_2(t) = \cos t + \cos(\sqrt{7}t)$

(h) $x_1(t) = 6\cos t + 4\cos(\sqrt{7}t), x_2(t) = 3\cos t - \cos(\sqrt{7}t)$

6.19(a) 4×5

(b) $\begin{pmatrix} 7 & p & 3 & 8 & 15 \end{pmatrix}$

(c) $\mathbf{M} = \begin{pmatrix} 3/2 & 1 & -2 & 9/2 & 0 \\ 3 & 5 & -3/2 & 6 & x/2 \\ 7/2 & p/2 & 3/2 & 4 & 15/2 \\ 1 & 23/2 & 3y/2 & 10 & f/2 \end{pmatrix}$

(d) 6

6.21(a) $\begin{pmatrix} 6 & 4 \\ 10 & -10 \end{pmatrix}$

(b) illegal

(c) illegal

(d) 37

6.23(a) $w = 0, x = -2, y = -6, z = -8$

(b) impossible

(c) impossible

6.25 $M_{ij} = i + 2j$

6.27(b) $\begin{matrix} meat \\ vegetables \end{matrix} \begin{pmatrix} 43/20 \\ 4/3 \end{pmatrix}$

6.29 $\begin{pmatrix} 68 + (2/3)x \\ -44 \end{pmatrix}$

6.31 (1)

6.33 Illegal

6.35(a) $\begin{matrix} Icelanders \\ Swedes \end{matrix} \begin{pmatrix} I \\ S \end{pmatrix}$

(b) $$\begin{array}{c} \\ O \\ A \\ B \\ AB \end{array} \begin{array}{c} \begin{array}{cc} \textit{Icelanders} & \textit{Swedes} \end{array} \\ \begin{pmatrix} .56 & .38 \\ .31 & .44 \\ .11 & .12 \\ .02 & .06 \end{pmatrix} \end{array}$$

(c) $$\begin{array}{c} O \\ A \\ B \\ AB \end{array} \begin{pmatrix} .56I + .38S \\ .31I + .44S \\ .11I + .12S \\ .02I + .06S \end{pmatrix}$$

6.37 $$\begin{array}{c} \textit{oribatid mites} \\ \textit{gamasid mites} \end{array} \begin{pmatrix} 1,736,000 \\ 1,074,000 \end{pmatrix} =$$

$$\begin{array}{c} \\ \textit{oribatid mites / } m^2 \\ \textit{gamasid mites / } m^2 \end{array} \begin{array}{c} \begin{array}{cc} \textit{forest floor} & \textit{grass clippings} \end{array} \\ \begin{pmatrix} 240,000 & 127,000 \\ 86,000 & 102,000 \end{pmatrix} \end{array} \begin{array}{c} \\ \textit{forest floor } (m^2) \\ \textit{grass clippings } (m^2) \end{array} \begin{pmatrix} 3 \\ 8 \end{pmatrix}$$

6.41(a) $$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2/3 & -1/3 \end{pmatrix} \begin{pmatrix} -10 \\ 3 \end{pmatrix}$$

(b) $$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} -7 \\ -23/3 \end{pmatrix}$$

6.45 $$\begin{array}{c} \hat{\imath} \\ \hat{\jmath} \end{array} \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

6.47(a) $$\begin{array}{c} \hat{\imath} \\ \hat{\jmath} \end{array} \begin{pmatrix} -24 \\ -3 \end{pmatrix} = \begin{array}{c} \\ \hat{\imath} \\ \hat{\jmath} \end{array} \begin{array}{c} \begin{array}{cc} \vec{A} & \vec{B} \end{array} \\ \begin{pmatrix} 1 & -4 \\ 2 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ \vec{A} \\ \vec{B} \end{array} \begin{pmatrix} a \\ b \end{pmatrix}$$

(b) $a = -4, b = 5$

6.49(a) $$\begin{pmatrix} 4x \\ -y \end{pmatrix}$$

6.51(a) **i** $$\begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

iii $$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

6.53(e) The natural basis consists of the vectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(f) $$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

6.55(a) $a = \dfrac{V_xB_y - V_yB_x}{A_xB_y - A_yB_x}, b = \dfrac{V_xA_y - V_yA_x}{A_xB_y - A_yB_x}$

(b) $B_y/B_x = A_y/A_x$

6.59(b) $\hat{\jmath}$

(c) $2\hat{\imath} + 1\hat{k}$

6.61(a) position and velocity

(b) 4

6.63(a) $\begin{matrix} x_1 \\ x_2 \end{matrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{matrix} x_1 \\ x_2 \end{matrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(b) $\begin{matrix} A \\ B \end{matrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{matrix} A \\ B \end{matrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

6.65 $x_1 = a + b, x_2 = (2a - b)/3$

6.67(a) $\mathbf{B} = \begin{matrix} & \textit{regular} & \textit{deluxe} \\ \textit{chocolates} \\ \textit{flowers} \\ \textit{cards} \end{matrix}\begin{pmatrix} 10 & 20 \\ 3 & 5 \\ 1 & 1 \end{pmatrix}$

(b) $\mathbf{S} = \begin{matrix} & \textit{drug} & \textit{grocery} \\ \textit{regular} \\ \textit{deluxe} \end{matrix}\begin{pmatrix} 20 & 50 \\ 10 & 30 \end{pmatrix}$

(c) $\mathbf{BS} = \begin{matrix} & \textit{drug} & \textit{grocery} \\ \textit{chocolates} \\ \textit{flowers} \\ \textit{cards} \end{matrix}\begin{pmatrix} 400 & 1,100 \\ 110 & 300 \\ 30 & 80 \end{pmatrix}$

(d) You have 8,200 chocolates, 2,240 flowers and 600 cards.

6.69(a) $\mathbf{F} = \begin{matrix} & \textit{young} & \textit{old} \\ \textit{sportscars} \\ \textit{minivans} \\ \textit{smartcars} \end{matrix}\begin{pmatrix} 2 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$

(b) $\mathbf{C} = \begin{matrix} & \textit{sportscars} & \textit{minivans} & \textit{smartcars} \\ \textit{cylinders} \\ \textit{seats} \end{matrix}\begin{pmatrix} 6 & 4 & 3 \\ 4 & 7 & 2 \end{pmatrix}$

(c) **CF**

(d) $\mathbf{CF} = \begin{matrix} & \textit{young} & \textit{old} \\ \textit{cylinders} \\ \textit{seats} \end{matrix}\begin{pmatrix} 12 & 13 \\ 8 & 13 \end{pmatrix}$

6.71 $\begin{pmatrix} 33 & 65 \\ 23 & 43 \end{pmatrix}$

6.73 $\begin{pmatrix} 0 & 6 & 7 & 0 \\ 0 & -11 & 0 & 0 \end{pmatrix}$

6.75 (44)

6.77 $\begin{pmatrix} 3 & 1 & 0 \\ 5 & 2 & 4 \\ 10 & 9 & 8 \end{pmatrix}$

6.79 $\begin{pmatrix} 0 & 1 & 3 \\ 4 & 2 & 5 \\ 8 & 9 & 10 \end{pmatrix}$

6.83(a) $\begin{pmatrix} 10 & -4 \\ 2 & 16 \\ 8 & 16 \end{pmatrix}$

(b) $\begin{pmatrix} 10 & -4 \\ 2 & 16 \\ 8 & 16 \end{pmatrix}$

6.85(b) $\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$

(c) stretches by 3 in the x-direction

(d) stretches by 3 in the y-direction

(e) stretches by 3 both in x and the y-directions

(f) diagonal stretch

6.89(a) $M_{11} = 2, M_{21} = 1$

(b) $M_{12} = -2, M_{22} = 2$

6.91(a) $\begin{pmatrix} 9 & 51 \\ -84 & 96 \\ -35 & 53 \end{pmatrix}$

(b) $\begin{pmatrix} 9 & 51 \\ -84 & 96 \\ -35 & 53 \end{pmatrix}$

6.93(a) $\mathbf{X} = \begin{array}{c} \\ \textit{regular} \\ \textit{edge} \end{array} \begin{array}{c} \begin{array}{cc} \textit{basic} & \textit{deluxe} \end{array} \\ \begin{pmatrix} 1,100 & 3,140 \\ 250 & 670 \end{pmatrix} \end{array}$

(b) $\begin{array}{c} \textit{regular} \\ \textit{edge} \end{array} \begin{pmatrix} 117,800 \\ 25,900 \end{pmatrix}$

(c) $\mathbf{X}^{-1} = \begin{pmatrix} -67/4800 & 157/2400 \\ 1/192 & -11/480 \end{pmatrix}$

(e) 52 basic, 73 deluxe

6.97(b) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}$

6.101(d) 4

6.103(a) **I**

(b) **X**

(c) **X**

6.105(a) $\begin{array}{c} \\ \textit{vampires} \\ \textit{zombies} \end{array} \begin{array}{c} \begin{array}{cc} \textit{swank} & \textit{trashy} \end{array} \\ \begin{pmatrix} 50 & 3 \\ 3 & 200 \end{pmatrix} \end{array}$

6.107(a) $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ -12 & 5 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 6 \\ 28 \end{pmatrix}$

(b) left

(c) $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$

(d) $\begin{pmatrix} -2/3 \\ 4 \end{pmatrix}$

(e) $x = -2/3, y = 4$

6.109 $x = -23/3, y = 10/9$

6.113(a) $\mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{D}$

(b) $\mathbf{D}\mathbf{C}^{-1}\mathbf{B}^{-1}$

(c) $\mathbf{A}^{-1}\mathbf{D}\mathbf{C}^{-1}$

6.115(a) $\begin{pmatrix} 1/3 & 1 \\ 2/3 & -1 \end{pmatrix}$

(c) $x_1(t) = -(1/3)\cos(\sqrt{2}t) + (25/3)\cos(\sqrt{5}t), x_2(t) = -(2/9)\cos(\sqrt{2}t) - (25/9)\cos(\sqrt{5}t)$

6.117(a) $\begin{array}{c} \\ \hat{\imath} \\ \hat{\jmath} \end{array} \begin{array}{c} \begin{array}{cc} \vec{P}_1 & \vec{P}_2 \end{array} \\ \begin{pmatrix} 3 & 2 \\ -4 & 1 \end{pmatrix} \end{array}$

(b) $\begin{array}{c} \\ \hat{\imath} \\ \hat{\jmath} \end{array} \begin{array}{c} \begin{array}{cc} \vec{Q}_1 & \vec{Q}_2 \end{array} \\ \begin{pmatrix} 1 & 0 \\ 5 & 4 \end{pmatrix} \end{array}$

(c) $\begin{array}{c} \\ \vec{Q}_1 \\ \vec{Q}_2 \end{array} \begin{array}{c} \begin{array}{cc} \hat{\imath} & \hat{\jmath} \end{array} \\ \begin{pmatrix} 1 & 0 \\ -5/4 & 1/4 \end{pmatrix} \end{array}$

(d) $\begin{array}{c} \\ \vec{Q}_1 \\ \vec{Q}_2 \end{array} \begin{array}{c} \begin{array}{cc} \vec{P}_1 & \vec{P}_2 \end{array} \\ \begin{pmatrix} 3 & 2 \\ -19/4 & -9/4 \end{pmatrix} \end{array}$

(e) $\begin{array}{c} \vec{Q}_1 \\ \vec{Q}_2 \end{array} \begin{pmatrix} 1 \\ -5/2 \end{pmatrix}$

6.119(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) Matrix $\mathbf{X}$ interchanges the axes. If you apply this transformation twice, you get your original points back. Every point (x, y) first becomes (y, x), then again (x, y).

6.123(a) $\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} w & y \\ x & z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) $aw + cx = 1, bw + dx = 0, ay + cz = 0, by + dz = 1$

(c) $w = d/(ad - bc), x = b/(bc - ad), y = c/(bc - ad), z = a/(ad - bc)$

(d) $\dfrac{1}{ad - bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

(e) $ad - bc = 0$

6.125(a) $x = y = 0$

(b) $x = y = 0$

(c) $x = y = 0$

(e) $\begin{pmatrix} 2 & 1 \\ 3 & 5 - \lambda \end{pmatrix}$

(f) $7 - 2\lambda$

(g) $7/2$

6.127 4

6.129 -46

6.131 unique solution, not homogeneous

6.133 inconsistent or linearly dependent, not homogeneous

6.135 unique solution, not homogeneous

6.139 -3

6.141 1 and 2

6.145(a) 12

(b) $aei + bfg + cdh - ceg - bdi - afh$

(c) $aei - ahf - bdi + bfg + cdh - ceg$

6.149(a) yes

(b) no

(c) no

6.153 $a = 2, b = 6, c = 1$

6.155 Inconsistent or linearly dependent.

6.157 45 mol/min C_2H_4 and 10 mol/min CH_4

6.159 Inconsistent or linearly dependent

6.161 $x_1 = A\cos(\sqrt{2}t) + B\cos(\sqrt{6}t)$, $x_2 = -(1/2)A\cos(\sqrt{2}t) + (1/2)B\cos(\sqrt{6}t)$

6.163(b) $50\vec{V}_1$

(c) $-10\vec{V}_2$

(d) $30\vec{V}_1 + 5\vec{V}_2$

(e) $5a\vec{V}_1 + b\vec{V}_2$

6.165(a) not an eigenvector

(b) $\lambda = 1$

(c) $\lambda = 5$

(d) not an eigenvector

6.171(a) $\lambda_1 = 4 + i$

(b) $\begin{pmatrix} 3-i \\ 5 \end{pmatrix}$

6.173(c) 2, 2, 1

6.175 $\vec{V}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ with $\lambda_1 = 10$, $\vec{V}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ with $\lambda_2 = -1$

6.177 $\vec{V}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ with $\lambda_1 = 8$, $\vec{V}_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ with $\lambda_2 = 1$

6.179 only one eigenvector

6.181(a) $\begin{pmatrix} 8 & 0 \\ 0 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 16/5 \\ -9/5 \end{pmatrix}$

6.183 $\begin{pmatrix} -146/7 & 130/7 \\ -156/7 & 153/7 \end{pmatrix}$

6.185(a) $\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = \begin{pmatrix} -8 & -8 \\ -1 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

(b) $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ for $\lambda = -4$, $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ for $\lambda = -10$.

(c) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A\cos(2t) + B\sin(2t) \\ (-1/2)A\cos(2t) - (1/2)B\sin(2t) \end{pmatrix}$

(d) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} C\cos(\sqrt{10}t) + D\sin(\sqrt{10}t) \\ (1/4)C\cos(\sqrt{10}t) + (1/4)D\sin(\sqrt{10}t) \end{pmatrix}$

(e) $x_1 = A\cos(2t) + B\sin(2t) + C\cos(\sqrt{10}t) + D\sin(\sqrt{10}t)$
$x_2 = (-1/2)A\cos(2t) - (1/2)B\sin(2t) + (1/4)C\cos(\sqrt{10}t) + (1/4)D\sin(\sqrt{10}t)$

(f) $x_1 = -\cos(2t) + 2\cos(\sqrt{10}t), x_2 = (1/2)\cos(2t) + (1/2)^{\rightarrow}\cos(\sqrt{10}t)$

6.187 $x_1 = -(1/9)e^{2t} + (10/9)e^{11t}$ and $x_2 = (1/3)e^{2t} + (5/3)e^{11t}$

6.189 $x_1 = -\sin(\sqrt{2}t) + (1/\sqrt{14})\left(e^{\sqrt{7}t} - e^{-\sqrt{7}t}\right), x_2 = (2/3)\sin(\sqrt{2}t)$
$+ 1/(3\sqrt{14})\left(e^{\sqrt{7}t} - e^{-\sqrt{7}t}\right)$

6.193(a) $\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} -6 & 1 \\ -4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(b) Eigenvector $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ has eigenvalue -2, eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ has eigenvalue -5.

(c) $\ddot{x} = -5x$ when $y = x$

(d) $y = x = A\cos(\sqrt{5}\,t) + B\sin(\sqrt{5}\,t)$

(e) $x = C\cos(\sqrt{2}\,t) + D\sin(\sqrt{2}\,t)$ and $y = 4x$

(f) $x = A\cos(\sqrt{5}\,t) + B\sin(\sqrt{5}\,t) + C\cos(\sqrt{2}\,t) + D\sin(\sqrt{2}\,t)$
$y = A\cos(\sqrt{5}\,t) + B\sin(\sqrt{5}\,t) + 4C\cos(\sqrt{2}\,t) + 4D\sin(\sqrt{2}\,t)$

6.195(a) $\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} -3 & -4 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(b) Eigenvector $\begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$ has eigenvalue -5, eigenvector $\begin{pmatrix} 1 \\ -3/2 \end{pmatrix}$ has eigenvalue 3.

(c) $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = f(t)\begin{pmatrix} 1 \\ 1/2 \end{pmatrix} + g(t)\begin{pmatrix} 1 \\ -3/2 \end{pmatrix}$

(d) $\ddot{f} = -5f, \ddot{g} = 3g$

(e) $f(t) = A\sin(\sqrt{5}t) + B\cos(\sqrt{5}t), g(t) = Ce^{\sqrt{3}t} + De^{-\sqrt{3}t}$

(f) $x(t) = A\sin(\sqrt{5}t) + B\cos(\sqrt{5}t) + Ce^{\sqrt{3}t} + De^{-\sqrt{3}t}, y(t) = (1/2)A\sin(\sqrt{5}t) + (1/2)B\cos(\sqrt{5}t) - (3/2)Ce^{\sqrt{3}t} - (3/2)De^{-\sqrt{3}t}$

6.197 $x(t) = A\sin(\sqrt{5}t) + B\cos(\sqrt{5}t) + Ce^{\sqrt{3}t} + De^{-\sqrt{3}t}, y(t) = (1/2)A\sin(\sqrt{5}t) + (1/2)B\cos(\sqrt{5}t) - (3/2)Ce^{\sqrt{3}t} - (3/2)De^{-\sqrt{3}t}$

6.199 $x(t) = (7/4)\cos t - (3/4)\cos(\sqrt{5}\,t), y(t) = -(7/2)\cos t - (3/2)\cos(\sqrt{5}\,t)$

6.201 $x(t) = (3/5)e^{6t} + (7/5)e^{t}, y(t) = (12/5)e^{6t} - (7/5)e^{t}$

6.205 $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \lambda = -1, -4$

6.207 $\dot{f} = -5f, \dot{g} = 2g$

6.209(a) $\dot{v} = -7x - 6y$

6.211 $\begin{pmatrix} A\sin(\sqrt{10}\,t) + B\cos(\sqrt{10}\,t) + C\sin(\sqrt{10/3}\,t) + D\cos(\sqrt{10/3}\,t) \\ -A\sin(\sqrt{10}\,t) - B\cos(\sqrt{10}\,t) + C\sin(\sqrt{10/3}\,t) + D\cos(\sqrt{10/3}\,t) \end{pmatrix}$

6.213 $\begin{pmatrix} A\sin(\sqrt{6}\,t) + B\cos(\sqrt{6}\,t) + C\sin(\sqrt{11}\,t) + D\cos(\sqrt{11}\,t) \\ 2A\sin(\sqrt{6}\,t) + 2B\cos(\sqrt{6}\,t) - (C/2)\sin(\sqrt{11}\,t) - (D/2)\cos(\sqrt{11}\,t) \end{pmatrix}$

6.215 $\begin{pmatrix} A\sin(\sqrt{12}\,t) + B\cos(\sqrt{12}\,t) + C\sin(\sqrt{10}\,t) + D\cos(\sqrt{10}\,t) + E\sin(2t) + F\cos(2t) \\ -A\sin(\sqrt{12}\,t) - B\cos(\sqrt{12}\,t) + 3E\sin(2t) + 3F\cos(2t) \\ A\sin(\sqrt{12}\,t) + B\cos(\sqrt{12}\,t) - C\sin(\sqrt{10}\,t) - D\cos(\sqrt{10}\,t) + E\sin(2t) + F\cos(2t) \end{pmatrix}$

6.217(a) $2\ddot{\theta}_1 + \ddot{\theta}_2 + 2(g/L)\theta_1 = 0, \ddot{\theta}_2 + \ddot{\theta}_1 + (g/L)\theta_2 = 0$

(b) $\ddot{\theta}_1 = -2(g/L)\theta_1 + (g/L)\theta_2, \ddot{\theta}_2 = 2(g/L)\theta_1 - 2(g/L)\theta_2$

(c) $\omega = \sqrt{(2 \pm \sqrt{2})(g/L)}$

6.219(b) In one normal mode $I_2 = I_1$ oscillating at a frequency $\sqrt{(1/L)(1/C_1 + 2/C_2)}$. In the other normal mode $I_2 = -I_1$ oscillating at frequency $\sqrt{1/LC_1}$.

6.221 $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

6.223(a) Yes

(c) $\begin{array}{c c} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} x \\ y \end{array} & \begin{pmatrix} -1 & 2 \\ -1 & 0 \end{pmatrix} \end{array}$

(d) $\begin{array}{c c} & \begin{array}{cc} x & y \end{array} \\ \begin{array}{c} a \\ b \end{array} & \begin{pmatrix} 0 & -1 \\ 1/2 & -1/2 \end{pmatrix} \end{array}$

(e) $\begin{pmatrix} -4 \\ -1/2 \end{pmatrix}$

6.225(a) $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} x/\sqrt{x^2+y^2} & -y/\sqrt{x^2+y^2} & 0 \\ y/\sqrt{x^2+y^2} & x/\sqrt{x^2+y^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

6.229(a) $I_3 = I_1 + I_2$

(e) $I_1(t) = (5/2)e^{-4\times10^{-6}t} + (1/2)e^{-2\times10^{-6}t}$, $I_2(t) = (5/2)e^{-4\times10^{-6}t} - (1/2)e^{-2\times10^{-6}t}$

Chapter 7

7.1(a) $\begin{pmatrix} 8 & 0 \\ 0 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & -3 \\ -8 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$

(f) $\begin{pmatrix} 0 & 8 \\ 3 & 0 \end{pmatrix}$

(g) $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

(h) $|\mathbf{M}| = |\mathbf{N}| = -24$

7.3(b) rotates 90° clockwise

(c) reflects about $y = x$

(d) reflects about the x-axis

(e) reflects about the y-axis

(f) reflects about the x-axis and stretches by a factor of 2 in the y-direction

(g) $|\mathbf{A}| = 1$, $|\mathbf{B}| = -1$, $|\mathbf{C}| = -2$

7.11(a) 2, with eigenvalues 1 and -1

(b) 3, with eigenvalues 1, 1, and -1

(c) 2, with eigenvalues 1 and -1

(d) 3, all with eigenvalue 1

(e) 2, both with eigenvalue 1

(f) 1, with eigenvalue -1

7.13(c) Matrix $\mathbf{M}$ stretches vectors by a factor of 6 in the $\hat{\imath} + 2\hat{\jmath}$ direction.

7.15 counterclockwise rotation of 30°

7.17 stretch by a factor of 6 along $6\hat{\imath} - 5\hat{\jmath}$, and 3 along $\hat{\imath} - \hat{\jmath}$

7.21 $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

7.23(a) $6\hat{\imath} - 15\hat{\jmath}$

(b) yes, 3.

(c) $6\hat{\imath} - 5\hat{k}$

(d) no

7.27(a) It's a cube of side length 1 with two opposite corners at the origin and (1, 1, 1).

(b) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & -1 & -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 0 & 0 \end{pmatrix}$

7.31(a) same orientation

(b) **i** same orientation

ii opposite orientation

iii opposite orientation

(c) $|\mathbf{T}_1| = 1, |\mathbf{T}_2| = 4, |\mathbf{T}_3| = -1, |\mathbf{T}_4| = -3$

7.41(a) $\begin{pmatrix} 5/4 & \sqrt{3}/4 \\ \sqrt{3}/4 & 7/4 \end{pmatrix}$

(b) $\begin{pmatrix} 1/\sqrt{3} \\ 1 \end{pmatrix}$ with eigenvalue 2, $\begin{pmatrix} -1 \\ 1/\sqrt{3} \end{pmatrix}$ with eigenvalue 1.

7.43 $\frac{1}{4}\begin{pmatrix} 13+5\sqrt{3} & -7+3\sqrt{3} \\ -3+3\sqrt{3} & 7-5\sqrt{3} \end{pmatrix}$

7.45(a) $\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 7/2 & -1/2 \\ -1/2 & 7/2 \end{pmatrix}$

7.49 $\begin{pmatrix} 1/2+\cos(\pi/9) & -1/2+\cos(\pi/9) & -\sin(\pi/9)/\sqrt{2} \\ -1/2+\cos(\pi/9) & 1/2+\cos(\pi/9) & -\sin(\pi/9)/\sqrt{2} \\ \sqrt{2}\sin(\pi/9) & \sqrt{2}\sin(\pi/9) & \cos(\pi/9) \end{pmatrix}$

7.51(a) $\vec{F}_g = -mg\hat{j}, \vec{F}_{fr} = |F_{fr}|\cos\theta\hat{i} + |F_{fr}|\sin\theta\hat{j},$
$\vec{F}_N = -|F_N|\sin\theta\hat{i} + |F_N|\cos\theta\hat{j}, \vec{F}_T = -|F_T|\cos 45°\hat{i} + |F_T|\sin 45°\hat{j}$

(b) $\vec{F}_g = -mg\sin\theta\hat{i}' - mg\cos\theta\hat{j}', \vec{F}_{fr} = |F_{fr}|\hat{i}', \vec{F}_N = |F_N|\hat{j}', \vec{F}_T = -|F_T|\cos(45° + \theta)\hat{i}' +$
$|F_T|\sin(45° + \theta)\hat{j}'$

(c) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

7.53(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) The identity matrix **I**

7.55(a) $\begin{pmatrix} 0 & 2ay \\ 0 & 0 \end{pmatrix}$

(b) $\mathbf{L}' = \frac{ay}{2}\begin{pmatrix} \sqrt{3} & 3 \\ -1 & -\sqrt{3} \end{pmatrix}$

(c) $\mathbf{L}' = \frac{a}{4}\begin{pmatrix} \sqrt{3}x' + 3y' & 3x' + 3\sqrt{3}y' \\ -x' - \sqrt{3}y' & -\sqrt{3}x' - 3y' \end{pmatrix}$

(d) $\vec{v}' = (a/8)\left(x'^2 + 2\sqrt{3}x'y' + 3y'^2\right)(\sqrt{3}\hat{i}' - \hat{j}')$

7.57(a) $\mathbf{C} = \begin{pmatrix} \gamma & -\gamma v/c^2 & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(b) $\mathbf{C}^{-1} = \begin{pmatrix} \gamma & \gamma v/c^2 & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

7.59(a) $\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(b) $\mathbf{M}' = \begin{pmatrix} 2 - 1/\sqrt{2} & (\sqrt{2}-1)/c & 1/(c\sqrt{2}) & 0 \\ (1-\sqrt{2})c & \sqrt{2}-1 & -1 & 0 \\ c/\sqrt{2} & 1 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

7.61(a) $\vec{r}_A = (t, 0, 0, 0), \vec{r}_B = (t, L, 0, 0)$

(b) γL

(c) $\Delta t' = \gamma\tau, \Delta x' = \gamma v \tau$

7.71 No

7.77(a) Hermitian but not unitary

(b) Unitary but not Hermitian

(c) Not Hermitian or unitary

7.79(a) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

(c) **i** Yes

ii No.

iii No.

iv No.

v No.

vi Yes.

7.81(b) $(1/\sqrt{2})(\hat{\imath} + \hat{\jmath})$

(c) $(1/\sqrt{11})(\hat{\imath} - \hat{\jmath} - 3\hat{k})$

(d) $(1/\sqrt{22})(-3\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

(f) $\vec{e}_1 = (1/\sqrt{11})(\hat{\imath} + \hat{\jmath} - 3\hat{k}), \vec{e}_2 = (1/\sqrt{990})(14\hat{\imath} + 25\hat{\jmath} + 13\hat{k})$

7.85(a) $\begin{pmatrix} 2 & 3 & 1 & 1 & 2 \\ -2 & -1 & 2 & 4 & -21 \\ 8 & 20 & 7 & 4 & 10 \\ 2 & 15 & 10 & 11 & -34 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 3 & 1 & 1 & 2 \\ 0 & 2 & 3 & 5 & -19 \\ 8 & 20 & 7 & 4 & 10 \\ 2 & 15 & 10 & 11 & -34 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 3 & 1 & 1 & 2 \\ 0 & 2 & 3 & 5 & -19 \\ 0 & 8 & 3 & 0 & 2 \\ 2 & 15 & 10 & 11 & -34 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 3 & 1 & 1 & 2 \\ 0 & 2 & 3 & 5 & -19 \\ 0 & 8 & 3 & 0 & 2 \\ 0 & 12 & 9 & 10 & -36 \end{pmatrix}$

(e) $\begin{pmatrix} 2 & 3 & 1 & 1 & 2 \\ 0 & 2 & 3 & 5 & -19 \\ 0 & 0 & -9 & -20 & 78 \\ 0 & 0 & -9 & -20 & 78 \end{pmatrix}$

(f) $\begin{pmatrix} 2 & 3 & 1 & 1 & 2 \\ 0 & 2 & 3 & 5 & -19 \\ 0 & 0 & -9 & -20 & 78 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

(h) Rank 3, linearly dependent equations, you could solve for three variables in terms of the fourth.

7.87 Rank 2, $x = 5/2$, $y = 1/2$

7.89 Rank 2, linearly dependent, you could solve for two variables in terms of the third.

7.91 Rank 3, Inconsistent

7.93 Rank 4, $x = -3$, $y = 4$, $z = 0$, $t = 8$

7.95 Rank 3, linearly dependent, we can solve for three variables in terms of the fourth.

7.97 Rank 4, $a = -3$, $b = 4$, $c = 1$, $d = 2$

7.99(b) $-3C + 2D + 4F = 2A,\ -2C + E + 2F = A,\ -5C + 4D + 6F = 2B,\ -7C + 4E + 6F = 2B$

(f) measure 3, calculate 3

7.103 $f = 2$

7.105 $f = 4$

7.107(a) 2

(b) 7 basic, 2 nonbasic

(c) 3

(d) m basic, 3 nonbasic

7.111 $f = 2$

7.113 $f = 1$

7.115 $f = 2$

7.117 $f = 2$

7.119 $f = -5/2$

7.121(a) $S_1 = 5, S_2 = 6, S_3 = 1$ and $f = 3$.

(b) $S_1 = 203, S_2 = 303, S_3 = 1$ and $f = 102$

(c) f increases without bound.

(d) S_3 becomes negative, which is not allowed.

7.125 $f = 3/2$

7.127 $f = 4$

7.129 $f = 12$

7.131 $f = 18$

7.137(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix}$

(f) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(h) Rotate 90° clockwise

(i) $-\hat{j}, -(1/2)\hat{i} - \hat{j}, -(1/2)\hat{i} + \hat{j}$

(j) $\begin{pmatrix} 0 & -1/2 \\ -1 & 0 \end{pmatrix}$

(k) $\begin{pmatrix} 0 & -1 \\ -2 & 0 \end{pmatrix}$

7.139(a) $\begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & -1 \\ -1 & -3 \end{pmatrix}$

(c) $\mathbf{B} = \mathbf{CA}$

(d) $\mathbf{C} = \mathbf{BA}^{-1}$

(e) $\mathbf{C} = \frac{1}{3}\begin{pmatrix} -1 & -5 \\ 5 & 1 \end{pmatrix}$

7.141(c) $\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \\ 1 & 1 & 1 \end{pmatrix}$

(d) adds X to all x coordinates, Y to y coordinates and Z to z coordinates.

(e) yes

7.143(a) $\frac{1}{2}\begin{pmatrix} \lambda_1 + \lambda_2 & \lambda_1 - \lambda_2 \\ \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 \end{pmatrix}$

(b) yes

(c) 2

7.147(a) $\mathbf{I} = \begin{pmatrix} 2/3 & -1/4 & -1/4 \\ -1/4 & 2/3 & -1/4 \\ -1/4 & -1/4 & 2/3 \end{pmatrix} ML^2$

(b) $\left[((2/3)a - (1/4)b)\,\hat{i} + (-(1/4)a + (2/3)b)\,\hat{j} + (-(1/4)a - (1/4)b)\,\hat{k}\right] ML^2$

Chapter 8

8.1(a) 1

(b) 2

(c) $\lim_{x\to\infty} \sigma(x) = \infty$

(d) $\lim_{x\to-\infty} \sigma(x) = \infty$

(e) -1

(f) $\lim_{y\to\infty} \sigma(x) = 0$

(g) $\lim_{y\to-\infty} \sigma(x) = \infty$

(j) $(16/3)(1 - 1/e)$

8.5(a) ii.

(c) force in the $-x$ direction

8.7(a) North

8.9(a) Northeast

(b) Southwest

8.13(a) planes

(b) planes

(c) Cylinders centered around the origin

(d) Spheres centered on the origin

(e) Ellipsoids centered on the origin, shorter in the z-direction

8.23(a) $2\pi r$

(b) $n/2\pi r$

8.29(a) negative

(b) positive

8.31 Speed up to the left

8.37(b) $x = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

(c) $x = 3\pi/2$ and $7\pi/2$

(d) $x = \pi/2$, and $5\pi/2$

8.51(a) left

(b) left

(d) positive charge left, negative charge right

8.53(a) $\vec{g} = \left(-GM/(x^2 + y^2 + z^2)^{3/2}\right)(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$

8.55 0

8.57 $\vec{0}$

8.59(a) In the direction of $3\hat{\imath} - 2\hat{\jmath}$ or about 34° below the positive x-axis

(b) $y = 3x + 1$

(c) neither up nor down

8.61(a) $\vec{\nabla}P(x, y, z) = -\hat{k}$

8.65(b) $V = k/\sqrt{x^2+y^2} - k/\sqrt{(x-5)^2+y^2}$

(c) $\vec{E} = \left[kx/(x^2+y^2)^{3/2} - k(x-5)/((x-5)^2+y^2)^{3/2}\right]\hat{\imath} + \left[ky/(x^2+y^2)^{3/2} - ky/((x-5)^2+y^2)^{3/2}\right]\hat{\jmath}$

8.69(a) $-1/3$

(b) $-4/3$

(c) $-(1/3)y^3 - x^3y$

(e) different by a constant

8.71 $F = -x^2 - \sin y$

8.73 $F = -\ln|x|$

8.75 $F = -\ln\left|3x + y^2\right|$

8.83(a) $2/(2x+3y) + 2x/z + (x+1)y$

(b) $5/3$

(d) $((x+1)z + 2xy/z^2)\hat{\imath} - yz\hat{\jmath} + (2y/z - 3/(2x+3y))\hat{k}$

(e) $2\hat{\imath} - 2\hat{\jmath}$

8.85 divergence=0; curl is out of the page

8.87 divergence is negative; curl=0

8.89 divergence is positive; curl=0

8.93 $\vec{\nabla}\cdot\vec{L} = 1$, $\vec{\nabla}\times\vec{L} = \vec{0}$

8.95 $\vec{\nabla}\cdot\vec{f} = 4$, $\vec{\nabla}\times\vec{f} = (3\pi - 48)\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

8.97 $\vec{\nabla}\cdot\vec{h} = 1/4$, $\vec{\nabla}\times\vec{h} = -404.75\,\hat{\jmath} + (3/16)\hat{k}$

8.99(b) 8

(c) -8

(d) 0

8.101(a) $\vec{E} = kQ(x/(x^2+y^2+z^2)^{3/2}\hat{\imath} + y/(x^2+y^2+z^2)^{3/2}\hat{\jmath} + z/(x^2+y^2+z^2)^{3/2}\hat{k})$

(c) 0

(e) 0

8.103 Laplacian is a scalar

8.109 positive: $|x| > |y|$; negative: $|x| < |y|$; 0: $x = \pm y$

8.111(a) $df_x/dx = 0$

(b) $dg_x/dx = 2$

(c) $dh_x/dx = -1/x^2$

8.115(a) $4\pi kq$

(b) $(3kq)/R^3$

(c) ∞

8.117(b) $<-\sqrt{3}/2, 1/2>$

(c) $<-\sqrt{3}/2, 1/2>$

8.119 $\vec{\nabla}\cdot\vec{f}(1,0) = 0$

8.121 $\vec{\nabla}\cdot\vec{f}(1,0) = 2$

8.123 $\vec{\nabla}f = (2+3\phi)\hat{\rho} + 3\hat{\phi}$, $\nabla^2 f = (2+3\phi)/\rho$

8.125 $\vec{\nabla}f = 2\rho\hat{\rho} + 2z\hat{k}$, $\nabla^2 f = 6$

8.127 $\vec{\nabla} f = (\theta + \phi)\hat{r} + \hat{\theta} + (1/\sin\theta)\hat{\phi}$, $\nabla^2 f = 2(\theta + \phi)/r + \cos\theta/(r\sin\theta)$

8.129 $\vec{\nabla} f = (\tan\theta/r)\hat{\theta} + \cos\phi/(r\sin\theta\sin\phi)\hat{\phi}$,
$\nabla^2 f = -1/(r^2\sin^2\theta\sin^2\phi) + 1/r^2 + 1/(r^2\cos^2\theta)$

8.131 $\vec{\nabla}\cdot\vec{f} = 1/\rho$, $\vec{\nabla}\times\vec{f} = 2\hat{k}$

8.133 $\vec{\nabla}\cdot\vec{f} = z^2/\rho + 2\rho\sin\phi\cos\phi$, $\vec{\nabla}\times\vec{f} = (2z - 2\rho)\hat{\phi} + 3\rho\sin^2\phi\,\hat{k}$

8.135 $\vec{\nabla}\cdot\vec{f} = 0$, $\vec{\nabla}\times\vec{f} = (\cot\theta)/r\ \hat{r} - (1/r)\hat{\theta}$

8.137 $\vec{\nabla}\cdot\vec{f} = 3\cos\theta + \cos(2\theta)\csc\theta$, $\vec{\nabla}\times\vec{f} = \cos(2\theta)\csc\theta\hat{r} - 2\cos\theta\,\hat{\theta} + (2\cos\theta + \sin\theta)\hat{\phi}$

8.141 div= 0; curl= $\vec{0}$.

8.143(a) $f_z(\rho, \phi, z + \Delta z) \times \Delta\rho(\rho\Delta\phi)$

(b) $-f_z(\rho, \phi, z) \times \Delta\rho(\rho\Delta\phi)$

(c) $\partial f_z/\partial z$

(d) $(1/\rho)(\partial f_\phi/\partial\phi)$

8.147 0

8.149 $3L^3$

8.151 $(1/3)\pi^4\,(1/e - e)$

8.153 $(8/3)\pi R^3$

8.155 $(12/5)\pi R^5$

8.157(a) $kD_0R^3(4\pi)$

8.159 2π

8.161 $-1/2$

8.163 $2\pi R\ln R$

8.165 0

8.167 $1/2$

8.173(d) $V(\rho) = -[\lambda/(2\pi\varepsilon_0)]\ln\rho$, $V(1) = 0$

8.175 (a),(c),(d)

8.177 conservative

8.179 not conservative

8.181 not conservative

8.183 conservative

8.187(a) $kq/\left(x^2 + y^2 + z^2\right)^{3/2}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

(b) $(kq/r^2)\hat{r}$

8.189(a) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I$

8.191 $\vec{\nabla}\cdot\vec{f} = 2(3x - y + z)$; $\vec{\nabla}\times\vec{f} = 2(y - z)\hat{i} - 6x\hat{k}$

8.193 $\vec{\nabla}\cdot\vec{f} = \sin\phi - \rho^2/z^2$; $\vec{\nabla}\times\vec{f} = -(2\rho/z)\hat{\phi} + \cos\phi\,\hat{z}$

8.195 $\vec{\nabla}\cdot\vec{f} = 3\sin\theta - \sin\phi/\sin\theta$; $\vec{\nabla}\times\vec{f} = (\cos\phi\cos\theta/\sin\theta)\,\hat{r} - 2\cos\phi\,\hat{\theta} - \cos\theta\,\hat{\phi}$

8.197 no solution

8.199 no solution

8.201 $F = y + ze^x$

8.203 $(4/3)\pi R^3 k$

8.205(a) 0

(b) 0

8.207(b) square in xy-plane

(c) 0

8.209 0

8.211 $-2/3$

8.213 $4\pi/3$

8.215(a) $3H\rho(t) = -d\rho/dt$

(b) 3 Billion years ago

(c) $\rho(t) = C/t^2$

(d) 4.1 Billion years ago

Chapter 9

9.1 $f(x) \approx 6\sin(2x) + \sin(4\pi x)$

9.3(a) -2

(b) 6

(c) $1/3$

(d) $-1/3$

(e) odd

(f) 8

(g) 4

(h) neither

9.5(c) neither

9.15 $\frac{1}{2} + \sum_{odd\ n} \frac{2}{n\pi}\sin(nx)$

9.17 $\pi^2 + \sum_{n=1}^{\infty} \frac{(-1)^n 12}{n^2}\cos(nx) + \sum_{n=1}^{\infty} \frac{(-1)^n 2}{n}\sin(nx)$

9.19 $\frac{e^{\pi} - e^{-\pi}}{\pi}\left[\frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2}(\cos(nx) - n\sin(nx))\right]$

9.23(a) $4\pi, 2\pi, 4\pi/3, \pi$

(b) 4π

9.27(a) $\frac{1}{4} + \frac{\sqrt{2}}{\pi}\cos(x) + \frac{1}{\pi}\cos(2x) + \frac{\sqrt{2}}{3\pi}\cos(3x) - \frac{\sqrt{2}}{5\pi}\cos(5x) - \frac{1}{3\pi}\cos(6x) - \frac{\sqrt{2}}{7\pi}\cos(7x)$

(b) $a + bS$ where $a = 1/4$ and $b = 1/\pi$

(c) $\pi/4$

9.29(a) odd

9.33(a) π

(b) π

(c) $1/2$ for both of them

9.35 $(1/\pi)\int_{-\pi}^{\pi} f(x)\cos(kx)dx$

9.37(a) 2π

(b) π

(c) 10π

(d) $2\pi/3$

(e) π

9.39(a) $\pi/3$

(b) $(2/3)\pi$

(c) period=$6/n, p = \pi n/3$ where n is any positive integer

(d) 6

9.41(a) period=6

(b) period=6

(c) period=3

(d) same periods

9.47 $\frac{e^5-1}{5}+\sum_{n=1}^{\infty}\frac{10(e^5-1)}{25+4n^2\pi^2}\cos\left(\frac{2n\pi}{5}x\right)+\sum_{n=1}^{\infty}\frac{4n\pi(1-e^5)}{25+4n^2\pi^2}\sin\left(\frac{2n\pi}{5}x\right)$

9.49 $\frac{3}{2}-\sum_{odd\ n}\frac{2}{n\pi}\sin\left(\frac{n\pi}{3}x\right)$

9.51 $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}4}{n\pi}\sin\left(\frac{n\pi}{2}x\right)$

9.53 Sine series: $\sum_{odd\ n}\frac{12}{n\pi}\sin(n\pi x)$, Cosine series: 3

9.59 $\sum_{n=1}^{\infty}\frac{27\sin(2n\pi/3)}{n^2\pi^2}\sin\left(\frac{n\pi}{65}x\right)$

9.61 $\sum_{odd\ n}-\frac{2i}{n\pi}e^{inx}$

9.63 $\frac{1}{2}+\sum_{odd\ n}\frac{2}{\pi^2n^2}e^{inx}$

9.65 $\frac{5}{2}+\sum_{n=-\infty}^{\infty}\frac{5i}{2n\pi}e^{i(2n\pi/5)x}$

9.67 $\frac{i}{3}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n(2+n\pi)i}{n^2\pi^2}e^{in\pi x}$

9.71(b) c_n is imaginary and $c_{-n}=c_n^*$.

9.73(a) $2A\cos(n\pi x/L)-2B\sin(n\pi x/L)$

(b) $2\sqrt{A^2+B^2}$

9.75(a) $c_n=(-1)^n i/n\ (n\neq 0)$, $c_0=0$

(b) $\sum_{n=-\infty}^{\infty}|c_n|^2=2\sum_{n=1}^{\infty}(1/n^2)$

(c) $\pi^2/3$

(d) $\pi^2/6$

9.77 $\pi^4/90$

9.89 $t_0/[2\pi(1+it_0\omega)]$

9.93(b) 1

(c) e^{ix}

9.99(a) $\hat{f}_0=-1, \hat{f}_1==8-3i, \hat{f}_2=-7$

(b) $8+3i$

(c) $\hat{f}_{-2}=-7, \hat{f}_3=8+3i$

(d) $\hat{f}_{-1}$ and $\hat{f}_1$ together represent the frequency $\pi/4$. $\hat{f}_0$ represents the constant term, and $\hat{f}_2$ the frequency $\pi/2$.

9.101 $(18, 10+6i, 2, 10-6i, 18)$, $p_{min}=5\pi/6$, $p_{max}=5\pi/3$

9.103 $(2, 2+2i, 14, 2-2i, 2)$, $p_{min}=\pi/2$, $p_{max}=\pi$

9.105 $(8, -4+4\sqrt{3}\,i, 8+4\sqrt{3}\,i, -4, 8-4\sqrt{3}\,i, -4-4\sqrt{3}\,i, 8)$, $p_{min}=\pi/12$, $p_{max}=\pi/4$

9.117 $f(x,y)=(5/2)+\sum_{n=-\infty}^{\infty}(2i/n\pi)e^{in\pi y}+\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(2/mn\pi^2)e^{im\pi x}e^{in\pi y}$ for odd m and n only

9.119 $$f(x) = \frac{2}{3} + \sum_{m=-\infty}^{\infty} \frac{2i}{3m\pi} e^{2m\pi ix} + \sum_{n=-\infty}^{\infty} \frac{1+in\pi}{n^2\pi^2} e^{n\pi iy} + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{i-n\pi}{n^2 m\pi^3} e^{2m\pi ix} e^{n\pi iy} \quad m, n \neq 0$$

9.121 $$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{(-1)^{m+n}(e^{4\pi} - e^{-4\pi})(e^{2\pi} - e^{-2\pi})}{4\pi^2(4-im)(2+in)} e^{m\pi ix/2} e^{n\pi iy/2}$$

9.123(a) $z = (2H/W)y$, $z = (2H/L)x$, $z = 2H(1 - y/W)$, $z = 2H(1 - x/L)$

(b) odd

9.129 $\sin(2x) + \cos(3x)$, $(i/2)e^{-2ix} - (i/2)e^{2ix} + (1/2)e^{3ix} + (1/2)e^{-3ix}$

9.133 $$\frac{1}{2}(1 - \cos 1) + \sum_{n=1}^{\infty} \frac{(\cos 2) - 1}{n^2\pi^2 - 1} \cos(2n\pi x) + \sum_{n=1}^{\infty} \left(\frac{8n}{\pi(4n^2 - 1)} - \frac{(\sin 2)n\pi}{n^2\pi^2 - 1} \right) \sin(2n\pi x),$$
$$\sum_{n=-\infty}^{\infty} \left(\frac{4in}{\pi(1 - 4n^2)} + \frac{\cos 2 + in\pi \sin 2 - 1}{2(n^2\pi^2 - 1)} \right) e^{2in\pi x}$$

9.137(b) $3 + \sum_{n=1}^{\infty} (6/n\pi) \sin(2n\pi x/3)$

(c) odd

(d) $3 - \sum_{n=-\infty}^{\infty} (3i/n\pi) e^{2in\pi x/3}$ $(n \neq 0)$

9.139(a) 2π

(b) 8π

(c) odd

(d) even

(e) $y(x, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} 16/(nm\pi^2) \sin(n\pi/2) \sin(mx) \cos(nt/4)$, odd m only

9.143 $$A \sin(2t) + B \cos(2t) + \frac{1}{4} + \sum_{odd\ n} \frac{4}{n\pi(4 - n^2\pi^2)} \sin(n\pi t)$$

Chapter 10

10.1(a) $(d^2y/dx^2) + 3(dy/dx) + 2y = 0$

(b) $y_c = C_1e^{-2x} + C_2e^{-x}$

(c) $2Ax^2 + (6A + 2B)x + (2A + 3B + 2C) = 6x^2$

(e) $y_p = 3x^2 - 9x + 21/2$

(f) $y(x) = 3x^2 - 9x + 21/2 + C_1e^{-2x} + C_2e^{-x}$

10.3(a) $Ny = -1/x^3$

(e) $y = \sqrt{x + C}$

10.5(d) $y(x) = -5\ln x/x + C_1x + C_2/x$

10.7 $y(x) = -5 + C_1e^{x/2} + C_2e^{-4x}$

10.9 $y(x) = -(1/3)xe^{-4x} + C_1e^{x/2} + C_2e^{-4x}$

10.11 $y(x) = 2e^{2x} + C_1\sin(3x) + C_2\cos(3x)$

10.13 $y(x) = (p/k)x^2 - 2(p/k^2)x + 2(p/k^3) + Ce^{-kx}$

10.15 $y(x) = 5x - 2 + C_1e^{-x}\cos(2x) + C_2e^{-x}\sin(2x)$

10.17 $y(x) = (ad/b^2) - (c/b) - (d/b)x + C_1e^{(1/2)[-a+\sqrt{a^2-4b}]x} + C_2e^{(1/2)[-a-\sqrt{a^2-4b}]x}$

10.19 $y(x) = -3 + C_1x^2 + C_2x^{-4/3}$

10.21 $y(x) = (27/10)e^{-x} - (17/325)\cos(4x) - (306/325)\sin(4x) + C_1e^{-6x} + C_2e^{-3x}$

10.25 linear

10.27 $x = -2e^{-t}\sin t$

10.29 $x = Ae^{p_1t} + Be^{p_2t}$ where $p = \dfrac{-b \pm \sqrt{b^2 - 4mk}}{2m}$

10.31(a) $I(t) = -0.059\cos(360t) + 0.072\sin(360t) + e^{-50t}\,[0.059\cos(312\,t) - 0.073\sin(312\,t)]$

10.35(a) $dy/dx = -2y/(x + y)$

(b) $-2mx/(x + mx) = m$

(c) $m = 0$ or $m = -3$

(d) $y = -3x$

10.37(a) right

10.51 $x'(t) = -f(x, y),\ y'(t) = -g(x, y)$

10.63(a) $dx/dy - (\sin x)y = \sin x$

(b) $-\sin x$

(c) $e^{\cos x}$

(d) $e^{\cos x}(dy/dx) - e^{\cos x}(\sin x)y = e^{\cos x}(\sin x)$

(f) $e^{\cos x}y(x) = -e^{\cos x} + C$

(g) $y(x) = Ce^{-\cos x} - 1$

10.65 $y(x) = (1/6)e^{3x} + Ce^{-3x}$

10.67 $y(x) = (e^x + C)/\sqrt{x}$

10.69 $y(x) = (x - e^{-x})/(e^x + 1) + C/(e^x + 1)$

10.71 $y = Ce^{-(2/3)x^3-3x} + 2$

10.73(b) $I(t) = V_0(5\sin(\omega t) - \omega\cos(\omega t))/(50 + 2\omega^2) + Ce^{-5t}$

10.77(a) $dM/dt = A + RM$

(b) $M(t) = -20,000 + Ce^{-e^{-t}/20}$

(c) $M(10) = 20,000\left[e^{(1-e^{-10})/20} - 1\right]$

10.83 $xy = C$

10.85 $3x^2 + 3x^2y + 10xy + 7y^2 = C$

10.87 $-y/(x+y) = C$

10.89(a) $dL_0\sqrt{1 - v^2/c^2}$

(b) $-vL_0/(c^2\sqrt{1 - v^2/c^2})\,dv$

(c) $(\sqrt{1 - v^2/c^2})\,dL_0 - (vL_0/(c^2\sqrt{1 - v^2/c^2}))\,dv = 0$

(d) $L_0\sqrt{1 - v^2/c^2} = C$

10.91 $V(x, y) = -e^{-x^2-y^2} - x + C$

10.93 D_1 is exact, with solution $xyzt + xe^z + t^2 = C$.

10.95 $y^2/2 - (3y + 2)/x = C$

10.97 $y^2e^{x+2y} = C$

10.99 $(x^2 + y)\left[\ln(x^2 + y) - 1\right] = C$

10.103 $N^{5/3}V^{-2/3}e^{(2/3)S/(Nk_B)} = C$

10.109 Linearly dependent

10.111 Linearly dependent

10.113 Linearly independent

10.115(a) $-4e^x \sin x$

(b) 0

(c) $-4e^{\pi/2}$

(e) no

10.117(a) $W'(x) = y_1y_2'' - y_1''y_2$

(b) $W'(x) = -a_1(y_1y_2' - y_1'y_2)$

(c) $W'(x) = -a_1W$

(d) $W(x) = Ce^{-\int a_1(x)dx}$

10.119(a) $f'(0) = 3g'(0)$

(c) $f''(0) = 3g''(0)$

10.121(a) $du/dt = 2t + 2$

(b) $dx/dt = (2t + 2)(dx/du)$

(d) $4u(d^2x/du^2) + 6u(dx/du) + 2ux = 0$

(e) $x(u) = Ae^{-u} + Be^{-u/2}$

(f) $x(t) = Ae^{-(t+1)^2} + Be^{-(t+1)^2/2}$

10.123 $y(x) = -x \pm \sqrt{2x + C}$

10.125 $y(x) = x^{-4}/(C - \ln x)$

10.127 $x(t) = \sqrt{A\cos(7t) + B\sin(7t)}$

10.129 $f(x) = A\cos x + B\cos x\ln(\cos x)$

10.131 $y(x) = x + 2/(3 + Ce^{6x})$

10.133 $y(x) = -(1/6)\ln(-6x + C) + x/3$

10.135 $x(t) = Ae^{-e^{2t}/2} + Be^{-e^{2t}}$

10.137 $x(t) = Ae^{\cos(2t)} + Be^{3\cos(2t)}$

10.139 $y(x) = x^2 - \sin^{-1}(-x + C)$

10.141(a) $k^2u^2(d^2x/du^2) + k^2u(dx/du) - 3ku(dx/du) + 36u^{6/k}x = 0$

(b) 3

(c) $x = A\cos(2u) + B\sin(2u)$

(d) $x(t) = A\cos(2e^{3t}) + B\sin(2e^{3t})$

10.143 $f(\theta) = AP_n(\cos\theta) + BQ_n(\cos\theta)$

10.145(a) $d^2\phi/dt^2 + (3/2)t^{-1}(d\phi/dt) + \lambda\phi^3 = 0$

(c) $k = 3/4$

(d) $u''(t) + (15/16)t^{-2}u + \lambda t^{-3/2}u^3 = 0$

10.147 $s = \dfrac{3}{2} \quad \rightarrow \quad a^{-3/2}\dfrac{d^2u}{dt^2} - \dfrac{3}{2}a^{-5/2}u\dfrac{d^2a}{dt^2} - \dfrac{3}{4}ua^{-7/2}\left(\dfrac{da}{dt}\right)^2 + a^{3/2}\dfrac{dV}{du} = 0$

10.151 $y(x) = [5/(18x^4 + Cx^9)]^{1/3}$

10.153 $y(x) = (x/3 + C/\sqrt{x})^2$

10.155 $y(x) = x/(C - \ln x)$

10.157 $y(x) = \pm x\sqrt{-\ln(-2\ln x + C)}$

10.159 $y(x) = \pm x\sqrt{\sin^{-1}(C + 2\ln x)}$

10.161 $y(x) = \pm 2/\sqrt{1 + Ce^{(8/3)x^3}}$

10.163 $y(t) = -\sin^{-1}(C - t) - t$

10.165 $x(t) = A\sin(1/x) + B\cos(1/x)$

10.167 $x(t) = A\sin t + B\cos t + Ct + D$

10.169(a) $dx/dt = -vx/\sqrt{x^2 + y^2}$

(b) $dx/dt = -vy/\sqrt{x^2 + y^2}$

(c) $dy/dx = y/x$

(d) $y = Cx$

(e) $dy/dx = \left(vy - s\sqrt{x^2 + y^2}\right)/(vx)$

(f) $dy/dx = \left(vy/x - s\sqrt{1 + (y/x)^2}\right)/v$

(g) $y = \left(x^{1-s/v} - L^{-2s/v}x^{1+s/v}\right)/2$

10.171(a) $v(x) = \sqrt{(2/3)gx + Cx^{-2}}$

(b) $v(x) = \sqrt{(2/3)gx}$

(c) $a(x, v) = \sqrt{g/6}\, x^{-1/2}v$

(d) $a = g/3$

10.173(a) $dy/dx = x(du/dx) + u$

(b) $x(du/dx) + u = f(u)$

(c) $du/(f(u) - u) = dx/x$

(d) $\int(du/(f(u) - u)) = \ln x + C$

10.177(b) $a_1(x) = 1/x^3 + 1/x,\ a_0(x) = -1/x^4 - 1/x^2$

(c) $u = -e^{1/(2x^2)}$

(d) $y_{c2}(x) = xe^{1/(2x^2)}$

(f) $y(x) = Ax + Bxe^{1/(2x^2)}$

10.181 $y(x) = Ax + B(x-1)e^{1/x}$

10.183(a) $y_{c1} = e^x$

(b) $3u' + 3u'' + u''' = 0$

(c) $u = \dfrac{e^{kx}}{k}$ where $k = \dfrac{-3 \pm \sqrt{-3}}{2}$

(d) $y = Ae^x + Be^{p_1 x} + Ce^{p_2 x}$ where $p_{1,2} = \dfrac{-1 \pm \sqrt{-3}}{2}$

10.185 $y(x) = -e^{5x}(1 + \ln x) + C_1 e^{5x} + C_2 x e^{5x}$

10.187 $y(x) = e^{4x}/14 + C_1 e^{x/2} + C_2 e^{2x}$

10.189 $y(x) = -x^2 - 1 + Ae^x + Bx$

10.191 $y(x) = x\cos x - x^2 \sin x + Ax + Bx \sin x$

10.195(a) $d^2x/dt^2 + 7(dx/dt) + 12x = F_e$

(b) $x_{1c} = e^{-3t}$ and $x_{2c} = e^{-4t}$

(c) $x_p(t) = e^{-3t} \int e^{3t} F_e(t)dt - e^{-4t} \int e^{4t} F_e(t)dt$

(d) **(i)** $x_p(t) = 5/12$

(ii) $x_p(t) = (8/63)e^{t/2}$

10.197(a) $b_{c1} = \sin(vn\pi t/L)$ and $b_{c2} = \cos(vn\pi t/L)$

(b) $\sin\left(\dfrac{vn\pi}{L}t\right) \displaystyle\int -\frac{4vL}{n^2\pi^2} \cos\left(\frac{vn\pi}{L}t\right) a(t)\, dt + \cos\left(\frac{vn\pi}{L}t\right) \int \frac{4vL}{n^2\pi^2} \sin\left(\frac{vn\pi}{L}t\right) a(t)\, dt$

(c) $b_{yn}(t) = -4kL^2/(n^3\pi^3)$

(d) $b_{yn}(t) = -t(4L^2/n^3\pi^3)$

10.209 $H(t-1) - H(t-4)$

10.211 $\sin(t)H(t-\pi)$

10.213 force over time

10.215 k/s

10.217 $1/s^2$

10.219 $(e^{-2s} - e^{-5s})/s$

10.221 $-e^{-s}(1/s + 1/s^2) + 1/s^2$

10.223 $e^{-3s} + e^{-4s}$

10.225 $(e^{-s} - e^{-4s})/s$

10.231(a) 1

(b) 0

(c) 4

(d) $4e^2$

10.233 $mv\delta(t)$

10.235(b) $t\,[H(t) - H(t-1)] + (2-t)\,[H(t-1) - H(t-2)]$

(c) $\left(e^{-2s} - 2e^{-s} + 1\right)/s^2$

10.237(a) $f(-50) = f(-5) = f(-1) = 0; f(1) = f(10) = f(100) = 1$

(c) Heaviside function

10.239 seconds^{-1}; thing·seconds

10.241(a) $1/(s-1)$

(c) $1/(s+1)$

(d) $1/(s^2-1)$

10.243(a) e^{-5s}

(b) $X(s) = (e^{-5s} + 6s + 23)/(3s^2 + 10s - 8)$

10.259 $x(t) = 13\cos t - 3\cos\left(\sqrt{11}\,t\right) - 3\sin t + 3\sqrt{11}\sin\left(\sqrt{11}\,t\right)$,
$y(t) = 39\cos t + \cos\left(\sqrt{11}\,t\right) - 9\sin t - \sqrt{11}\sin\left(\sqrt{11}\,t\right)$

10.261 e^{7t}

10.263 $\cos(5t)$

10.265(a) e^{-s}

(e) $x(t) = \left(1/\sqrt{2}\right)\sin\left(\sqrt{2}\,t\right)H(t)$

10.271 $I(s) = 9s(1 - e^{-2\pi s})/[(1+s^2)(2s^2+7s+6)]$

10.273(a) $-kx_2$

(b) $x_2 - x_1$

(c) Left mass: $k(x_2 - x_1)$. Right mass: $-k(x_2 - x_1)$.

(d) $mx_1'' = kx_2 - 2kx_1$, $mx_2'' = kx_1 - 2kx_2$

(e) $X_1 = 20s/(3s^4 + 40s^2 + 100)$, $X_2 = 2s(3s^2+20)/(3s^4+40s^2+100)$

10.275(a) $X(s) = F(s)/(s^2 + a_1 s + a_0)$

(b) $G(s) = 1/(s^2 + a_1 s + a_0)$

(c) $x''(t) + 2x'(t) + 3x(t) = f(t)$; $G(t) = 1/(s^2 + 2s + 3)$

10.277(a) $\mathcal{L}\left[tf(t)\right] = -F'(s)$

(b) $\mathcal{L}[ty'(t)] = -sY'(s) - Y(s)$

(c) $\mathcal{L}[ty''(t)] = -s^2Y'(s) - 2sY(s) + y(0)$

(d) $Y'(s)\left(-s^2 + s\right) + Y(s)\left(-s + 1 + n\right) = 0$

10.279(a) $f(x) = \int_0^\infty f(p)\delta(p-x)\,dp$

(b) $G''(x) + 2G'(x) - 15G(x) = \delta(x-p)$

(c) $G(x,p) = \begin{cases} Ae^{-5x} + Be^{3x} & 0 \le x < p \\ Ce^{-5x} + De^{3x} & x > p \end{cases}$

(d) $A + B = 0$

(e) $D = 0$

(f) $Ae^{-5p} + Be^{3p} = Ce^{-5p}$

(g) $-5Ae^{-5p} + 3Be^{3p} + 1 = -5Ce^{-5p}$

(h) $G(x,p) = \dfrac{1}{8}\begin{cases} (e^{-5x} - e^{3x})e^{-3p} & 0 \le x < p \\ e^{-5x}(e^{-3p} - e^{5p}) & x > p \end{cases}$

(i) $y(x) = (1/8)\left\{e^{-5x}\int_0^x (e^{-3p} - e^{5p})f(p)dp + (e^{-5x} - e^{3x})\int_x^\infty e^{-3p}f(p)dp\right\}$

(j) $y(x) = (1/16)(e^{-5x} - e^{-x})$

10.281 $y(x) = (1/6)x^3 - (50/3)x$

10.283 $y(x) = -(1/8)\left(e^{-5x}\int_{-\infty}^x e^{5p}f(p)dp + e^{3x}\int_x^\infty e^{-3p}f(p)dp\right)$

10.285 $G(x,p) = \begin{cases} -(\cos p)\sin x & x < p \\ -(\sin p)\cos x & x > p \end{cases}$

10.287 $y(x) = \cos x\ln(\cos x) + (x - \pi/2)\sin x$

10.289(b) $x(t) = -(1/4) + (1/36)e^{4t} + (2/9)e^{-t/2}$

10.291(b) $y_1 = A \sin x$

(c) $y_3 = D \cos x$

(d) $y_2 = 1/(2\epsilon) + B \sin x + C \cos x$

(f) $A = -\sqrt{3}/(2\epsilon) \sin \epsilon$, $D = -1/(2\epsilon) \sin \epsilon$

(g) $\Delta y = -(1/2)(\sin^2 \epsilon/\epsilon)$, $\Delta y' = (\sqrt{3}/2)(\sin^2 \epsilon/\epsilon) + \sin \epsilon \cos \epsilon/\epsilon$

(h) $\lim_{\epsilon \to 0} \Delta y = 0$, $\lim_{\epsilon \to 0} \Delta y' = 1$

10.293(a) $x''(t) + (b/m)x'(t) = F_{ext}(t)/m$

(b) $G(t,p) = (m/b)\left[1 - e^{(b/m)(p-t)}\right]$ for $t > p$

(c) For $t < t_1$, $x(t) = 0$. For $t_1 < t < t_2$, $x(t) = -(km/b^2)(t - t_1) + k/(2b)(t - t_1)^2 + km^2/(b^3)\left(1 - e^{-(b/m)(t-t_1)}\right)$. For $t > t_2$, $x(t) = -km/(b^2)(t_2 - t_1)e^{(b/m)(t_2-t)} + k/(2b)(t_2 - t_1)^2 + km^2/(b^3)\left(e^{(b/m)(t_2-t)} - -e^{-(b/m)(t-t_1)}\right)$

10.295 $f(t) = (1/4) + 2e^{-t^4}$

10.297 $f(t) = (1/50)\left(e^{5t} + 9\sin(5t) + 49\cos(5t)\right)$

10.299 $f^3 t + (1/3)t^3 + f^2 = C$

10.301 $f = (\sin^2 t + C)/(2\cos t)$

10.303 $e^{f^2+t} + \left(f^2 + t\right)^2 - t^2 = C$

10.305 $fe^t - (1/4)t^4 + (1/3)f^3 = C$

10.307 The Laplace transform of the solution is $F(s) = (2 + e^{-s})/(s^2 + 4s + 3)$.

10.309 $f(t) = -2 + C_1 t^{-2} + C_2 t^2$

10.311 $(f/t) + \ln(f/t) = \ln t + C$

10.313 $f(t) = (1/3)\cos^4 t + \sin^2 t - (1/3)\sin^4 t + A\cos t + B\sin t$

10.315 The Laplace transform of the solution is $s/(s+1)^2$

10.317 $y(x) = \ln(Ae^x + Be^{-x} + 1)$

10.319 $y(x) = -(1/C_1)\ln\left(1 - C_1 x^2\right) + C_2$

10.321 $y(x) = \pm x\sqrt{2\ln x + C}$

10.323(a) $x(t) = Ae^{-3t} + Bte^{-3t}$

(c) $x(t) = Ae^{-3t} + Bte^{-3t} + e^{-3t}\left(t\int e^{3t}F_e(t)dt - \int te^{3t}F_e(t)dt\right)$

(d) $x(t) = Ae^{-3t} + Bte^{-3t} + t^7 e^{-3t}/42$

10.325(a) $x(t) = (4/15)e^{-t} - (1/15)e^{-4t} - (1/5)\cos(2t)$

10.327(b) $\left(e^{-2s} - e^{-(2+a)s}\right)/as$

(c) e^{-2s}

(d) $\delta(t-2)$

10.331(b) $-x/y$

(c) $dy/dx = y/x$

(d) $y(x) = Cx$

(e) $y(x) = Cx^4$

10.333(a) $y(x) = 1/(8kx^2) + C$

(b) $m_{eq} = 4y/x$

(c) $y(x) = \pm\sqrt{C - x^2/4}$

Chapter 11

11.1 (b), (c)

11.3 (c), (d)

11.5 (a), (b)

11.7 (c)

11.9(d) Yes

(e) Yes

11.11(c) Yes

(d) No

11.23 $z(3, y) = 0$ is the only homogeneous condition

11.25 homogeneous

11.29 $\partial u/\partial t = \alpha\left(\partial^2 u/\partial x^2\right) + kx$

11.31(a) $\partial u/\partial t = \alpha\nabla^2 u$

(b) $\nabla^2 u_0 = 0$

11.33 $\partial^2 y/\partial t^2 = v^2\left(\partial^2 y/\partial x^2\right) - k^2\left(\partial y/\partial t\right)$

11.35(a) $\partial\rho/\partial t = k\left(\partial^2\rho/\partial x^2\right)$

11.45(a) $T_R = T\sin(\theta_R)$

(b) $T_R = T\sin\tan^{-1}(\partial y/\partial x)_R$

(c) $T_R \approx T\left(\partial y/\partial x\right)_R$

(d) $T_L \approx -T\left(\partial y/\partial x\right)_L$

(e) $T\left(\partial y/\partial x\right)_R - T\left(\partial y/\partial x\right)_L$

(f) $\lambda\, dx$

(g) $[T\left(\partial y/\partial x\right)_R - T\left(\partial y/\partial x\right)_L]/(\lambda\, dx)$

(h) $v = \sqrt{T/\lambda}$

11.51(a) $y(x, t) = \sin(5x)\cos(5vt)$

(c) $y(x, t) = 3\sin(2x)\cos(2vt)$

(d) $y(x, t) = 6\sin(x)\cos(vt) - 5\sin(7x)\cos(7vt)$

(e) $y(x, t) = \sum_{n=1}^{\infty}(1/n^2)\sin(nx)\cos(nvt)$

11.55 $y(x, t) = \sin(2x)\cos(4t) + (1/10)\sin(10x)\cos(20t)$

11.57 $y(x, t) = \sum_{n=1}^{\infty}(2/\pi n)\left[\cos(\pi n/3) - \cos(2\pi n/3)\right]\sin(nx)\cos(2nt)$

11.63 $y(x, t) = \sin(x)\cos(vt)$

11.65 $s(x, t) = \cos(x)\cos(3t) + (1/10)\cos(10x)\cos(30t)$

11.69(a) $y(x, 0) = \sin(n\pi x/L)$ where n is any integer

(b) $y(x, t) = \sin(n\pi x/L)\cos(nv\pi t/L)$

(c) $y(x, t) = \sin(3\pi x/L)\cos(3v\pi t/L)$

(d) $y(x, t) = 2\sin(3\pi x/L)\cos(3v\pi t/L) + 5\sin(8\pi x/L)\cos(8v\pi t/L)$

(e) $y(x, t) = \sum_{n=1}^{\infty}(2/\pi n)\left[\cos(\pi n/3) - \cos(2\pi n/3)\right]\sin(n\pi x/L)\cos(nv\pi t/L)$

11.71 $y(x, t) = \sin(kx + \phi)\cos(kvt)$ where k and ϕ are any real numbers.

11.75(a) $(1/2)\left[\sin(n(x + vt)) + \sin(n(x - vt))\right]$

(b) $f(x) = g(x) = \sum_{n=0}^{\infty}(b_n/2)\sin(nx)$

11.77 $y(x,t) = F\sin(2\pi x/L)\cos(2v\pi t/L)$

11.79 $y(x,t) = H\sin(2\pi x/L)\cos(2v\pi t/L) + (cL/v\pi)\sin(\pi x/L)\sin(v\pi t/L)$

11.81 $y(x,t) = \sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{L}x\right)\left[C_n\sin\left(\frac{nv\pi}{L}t\right) + D_n\cos\left(\frac{nv\pi}{L}t\right)\right]$ where

$C_n = \frac{2cL}{n^2\pi^2 v}\left[\cos\left(\frac{n\pi}{3}\right) - \cos\left(\frac{2n\pi}{3}\right)\right]$ and $D_n = \frac{4L}{\pi^2 n^2}\sin\left(\frac{n\pi}{2}\right)$

11.83(a) $D_n = (2/n\pi)\left[\cos(n\pi/3) - \cos(2n\pi/3)\right]$

(b) $y(x,t) = \sum_{n=1}^{\infty}(2/n\pi)\left[\cos(n\pi/3) - \cos(2n\pi/3)\right]\sin(n\pi x)\cos\left(\sqrt{n^2\pi^2+1}\,t\right)$

11.85 $y(x,t) = \sin(\pi x/L)\,e^{-c^2\pi^2 t/L^2}$

11.87 $y(x,t) = \sum_{n=1}^{\infty} D_n\sin(n\pi x/L)\cos\left(\sqrt{1+c^2n^2\pi^2/L^2}\,t\right)$ where

$D_n = (2/L)\int_0^L f(x)\sin(n\pi x/L)\,dx.$

11.89 $y(x,t) = \frac{1}{2}\left(\frac{L}{3\pi c}\right)^{3/2}\sin\left(\frac{3\pi}{L}x\right)\left[\sinh\left(\sqrt{\frac{3\pi c}{L}}\,t\right) - \sin\left(\sqrt{\frac{3\pi c}{L}}\,t\right)\right]$

11.91 $u(x,t) = u_0\sin(2\pi x/L)\,e^{-4\alpha\pi^2 t/L^2}$

11.93 $u(x,t) = \sum_{n=1}^{\infty}(2u_0/n\pi)(\cos(n\pi/3) - \cos(2n\pi/3))\sin(n\pi x/L)e^{-\alpha n^2\pi^2 t/L^2}$

11.97(a) $y(x,t) = 0$

(c) $X''(x) = PX(x),\ T''(t) = v^2PT(t)$

11.99(a) $-(\hbar^2/2m)(\psi''/\psi) + V(x) = i\hbar(T'/T) = E$

(b) $T = Ae^{-Eit/\hbar}$

(c) $E = n^2\pi^2\hbar^2/\left(2mL^2\right)$ for $n = 1, 2, 3\ldots$

(d) $\sum_{n=1}^{\infty} A_n\sin\left(\frac{n\pi}{L}x\right)$

(e) $\sum_{n=1}^{\infty} A_n\sin\left(\frac{n\pi}{L}x\right)e^{-n^2\pi^2\hbar it/(2mL^2)}$

(f) $\sum_{n=1}^{\infty}\frac{2\psi_0}{n\pi}\left[\cos\left(\frac{n\pi}{4}\right) - \cos\left(\frac{3n\pi}{4}\right)\right]\sin\left(\frac{n\pi}{L}x\right)e^{-n^2\pi^2\hbar it/(2mL^2)}$

11.103(a) $V(x,y,z) = [\sinh(\sqrt{5}\pi z/L)/\sinh(\sqrt{5}\pi)]\left[\sin(\pi x/L)\sin(2\pi y/L)\right.$
$\left.+\sin(2\pi x/L)\sin(\pi y/L)\right]$

11.107(b) $F_{nm} = (4/mn\pi^2)[\cos(2m\pi/3) - \cos(m\pi/3)][\cos(2n\pi/3) - \cos(n\pi/3)]$

11.109 $u(x,y,t) = \sin(2\pi x/L)\sin(3\pi y/H)e^{-\pi^2\left[4(a/L)^2+9(b/H)^2\right]t}$

11.111 $u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin(m\pi x/L)\sin(n\pi y/H)\sin(\sqrt{(m\pi/L)^2+(n\pi/H)^2+1}\,t)$
where $A_{mn} = (4/LH)[\int_0^H\int_0^L g(x,y)\sin(m\pi x/L)\sin(n\pi y/H)\,dx\,dy]/$
$\sqrt{(m\pi/L)^2+(n\pi/H)^2+1}$

11.113 $z(x,y,t) = \sin(\pi x/L)\sin(\pi y/L)\cos\left(\sqrt{2}v\pi t/L\right)$

11.117 $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}C_{mn}\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}y\right)\sin\left(\frac{\pi}{L}\sqrt{v_x^2m^2+v_y^2n^2}\,t\right)$

$C_{mn} = \frac{4}{\pi L\sqrt{v_x^2m^2+v_y^2n^2}}\int_0^L\int_0^L g(x,y)\sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{n\pi}{L}y\right)\,dx\,dy$

11.119 $y = AJ_2(x^2) + BY_2(x^2)$

11.121 $y = AJ_0(e^x) + BY_0(e^x)$

11.129 $z(\rho,t) = \sum_{n=1}^{\infty} \frac{2c}{a\alpha_{0,n} v J_1^2(\alpha_{0,n})} \left(\int_0^a \sin\left(\frac{3\pi}{a}\rho\right) J_0\left(\frac{\alpha_{0,n}}{a}\rho\right) \rho\, d\rho \right) J_0\left(\frac{\alpha_{0,n}}{a}\rho\right) \cos\left(\frac{\alpha_{0,n}}{a} vt\right)$

11.131 $z(\rho,\phi,t) = \sum_{n=1}^{\infty} \left[\frac{2}{a^2 J_2^2(\alpha_{1,n})} \int_0^a (a-\rho) J_1\left(\frac{\alpha_{1,n}}{a}\rho\right) \rho\, d\rho \right] J_1\left(\frac{\alpha_{0,n}}{a}\rho\right) \cos\left(\frac{\alpha_{0,n}}{a} vt\right) \sin(\phi)$

11.135(a) $T'(t)/T(t) = x^{3/2}(X''(x)/X(x)) + x^{1/2}(X'(x)/X(x))$

(b) $X(x) = AJ_0\left(4kx^{1/4}\right) + BY_0\left(4kx^{1/4}\right)$

(c) $X(x) = AJ_0\left(4kx^{1/4}\right)$

(d) $k = \alpha_{0,n}/4$

(e) $T(t) = Ce^{-\alpha_{0,n}^2 t/16}$

(f) $y(x,t) = \sum_{n=1}^{\infty} A_n J_0(\alpha_{0,n} x^{1/4}) e^{-\alpha_{0,n}^2 t/16}$

(g) $A_n = 1/(2J_1^2(\alpha_{0,n})) \int_0^1 \sin(\pi x) J_0(\alpha_{0,n} x^{1/4}) x^{-1/2} dx$

11.137 $y(x,t) = \sum_{n=1}^{\infty} A_n J_1\left(\alpha_{1,n} x/3\right) e^{-\alpha_{1,n}^2 t/9}$ where $A_n = (2/9J_2^2(\alpha_{1,n})) \int_1^2 J_1\left(\alpha_{1,n} x/3\right) x dx$

11.139 $z(x,y,t) = J_{2\pi}(\alpha_{2\pi,3} x) \sin(2\pi y) e^{-\alpha_{2\pi,2}^2 t}$

11.141(a) $T(t) = A\sin(kt) + B\cos(kt)$

(b) $c = 2k/\sqrt{g}$.

(d) $u(y,t) = \sum_{n=1}^{\infty} J_0\left(\sqrt{\frac{y}{L}}\alpha_{0,n}\right) \left[A_n \sin\left(\frac{\alpha_{0,n}\sqrt{g}}{2\sqrt{L}} t\right) + B_n \cos\left(\frac{\alpha_{0,n}\sqrt{g}}{2\sqrt{L}} t\right) \right]$

where $A_n = \frac{2}{\alpha_{0,n}\sqrt{g}\sqrt{L} J_1^2(\alpha_{0,n})} \int_0^L h(y) J_0\left(\frac{\alpha_{0,n}}{\sqrt{L}}\sqrt{y}\right) dy$ and

$B_n = \frac{1}{L J_1^2(\alpha_{0,n})} \int_0^L f(y) J_0\left(\frac{\alpha_{0,n}}{\sqrt{L}}\sqrt{y}\right) dy$

11.143 $t = a^2 \ln 2 / \left(D\alpha_{2,3}^2\right)$

11.145 $A_n = \frac{2c}{a\alpha_{0,n}^2 J_1(\alpha_{0,n})}$ and $B_n = \frac{2}{a^2 J_1^2(\alpha_{0,n})} \int_0^a (a-\rho) J_0\left(\frac{\alpha_{0,n}}{a}\rho\right) \rho d\rho$

11.149(a) $(1-x^2)(X''(x)/X(x)) - 2x(X'(x)/X(x)) = T'(t)/T(t)$

(b) $X(x) = AP_l(x)$

(c) $T(t) = Ce^{-l(l+1)t}$

(d) $y(x,t) = \sum_{l=0}^{\infty} A_l P_l(x) e^{-l(l+1)t}$

(e) $A_l = [(2l+1)/2] \int_{-1}^1 x P_l(x) dx$

(f) $y(x,t) = xe^{-2t}$

11.151 $y(x,t) = \sum_{l=0}^{\infty} A_l P_l(x/3) e^{-l(l+1)t}$ where $A_l = [(2l+1)/2] \int_{-1}^1 P_l(u) \sin(\pi u) du$

11.153 $4x\cos(\sqrt{2}\, t)$

11.155 $y(x,\phi,t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm} e^{-l(l+1)t} Y_l^m(\cos^{-1} x, \phi)$

where $C_{lm} = -\int_0^{2\pi} \int_{-1}^1 \left(1-x^2\right) \cos(\phi) \left[Y_l^m(\cos^{-1} x, \phi)\right]^* dx\, d\phi$

11.157(a) $R(r) = Ar^l + Br^{-l-1}$

11.159(a) $V(r,\theta,\phi) = V_0$

(b) $\vec{0}$

11.161(c) $u(r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} A_{lm}Y_l^m(\theta,\phi)j_l(\lambda r)$ where $\lambda = \alpha_{l+1/2}/a$

11.163(h) $V(r,\theta) = \sum_{l=0}^{\infty} C_l(r/a)^l P_l(\cos\theta)$ where $C_l = (2l+1)\int_0^1 cP_l(u)du$ (odd l only)

11.165(a) $f_1 = [3/\sin(7\pi/\sqrt{2})]\sin(7x/\sqrt{2})\sin(7y)$

(b) $f_2 = [4/\sin(5\sqrt{2}\,\pi)]\sin(5x)\sin(5\sqrt{2}\,y)$

11.167 $u_0\dfrac{e^{\mu(y-L)} - e^{-\mu(y-L)}}{e^{-\mu L} - e^{\mu L}}\sin\left(\dfrac{2\pi}{L}x\right) + \displaystyle\sum_{n=1}^{\infty}\dfrac{4u_0}{n\pi}\dfrac{e^{\nu x} - e^{-\nu x}}{e^{\nu L} - e^{-\nu L}}\sin\left(\dfrac{n\pi}{L}y\right)$ for odd n only, $\mu = \sqrt{\gamma^2 + 4\pi^2/L^2}$ and $\nu = \sqrt{\gamma^2 + n^2\pi^2/L^2}$

11.169 $\sin(\pi x)e^{(y-3)/2}\dfrac{e^{(1/2)\sqrt{4\pi^2+1}\;y} - e^{-(1/2)\sqrt{4\pi^2+1}\;y}}{e^{(3/2)\sqrt{4\pi^2+1}} - e^{-(3/2)\sqrt{4\pi^2+1}}}$
$+3\sin(2\pi x)e^{y/2}\dfrac{e^{(1/2)\sqrt{1+16\pi^2}\;y} - e^{3\sqrt{16\pi^2+1}}e^{-(1/2)\sqrt{1+16\pi^2}\;y}}{1 - e^{3\sqrt{16\pi^2+1}}}$

11.171(a) $(e^x - e^{-x})/(e - e^{-1})$

(b) $\sum_{n=1}^{\infty} A_n\sin(n\pi x)e^{-(1+n^2\pi^2)t}$

(c) $(e^x - e^{-x})/(e - e^{-1}) + \sum_{n=1}^{\infty} A_n\sin(n\pi x)e^{-(1+n^2\pi^2)t}$

(d) $\dfrac{e^x - e^{-x}}{e - e^{-1}} + \displaystyle\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}2}{n\pi(1+n^2\pi^2)}\sin(n\pi x)e^{-(1+n^2\pi^2)t}$

(e) $(e^x - e^{-x})/(e - e^{-1})$

11.173 $\alpha\sin\theta + \alpha\sin(4\theta)e^{-15t}$

11.175(b) $u(x,t) = 100x + \sum_{n=1}^{\infty}(800/n^3\pi^3)e^{-\alpha n^2\pi^2 t}\sin(n\pi x)$, odd n only

(c) 3996 s

11.179 $\displaystyle\sum_{n=1}^{\infty}\dfrac{4\kappa v^2L^2}{n\pi(v^2n^2\pi^2 - \omega^2L^2)}\left[\dfrac{L\omega}{n\pi v}\sin\left(\dfrac{n\pi v}{L}t\right) - \sin(\omega t)\right]\sin\left(\dfrac{n\pi}{L}x\right)$, odd n only

11.181 $\dfrac{\kappa v^2L^2}{p^2\pi^2v^2 + \omega^2L^2}\left[-e^{-\omega t} - \dfrac{\omega L}{p\pi v}\sin\left(\dfrac{p\pi v}{L}t\right) + \cos\left(\dfrac{p\pi v}{L}t\right)\right]\sin\left(\dfrac{p\pi}{L}x\right)$

11.183 $\displaystyle\sum_{n=1}^{\infty}\left[A_n\sin\left(\dfrac{n\pi v}{L}t\right) + B_n\cos\left(\dfrac{n\pi v}{L}t\right) - \dfrac{4\kappa v^2L^2}{n\pi\left(\omega^2L^2 + v^2n^2\pi^2\right)}e^{-\omega t}\right]\sin\left(\dfrac{n\pi}{L}x\right)$ for odd n only,
where $A_n = -\dfrac{4\kappa vL^3\omega}{n^2\pi^2\left(\omega^2L^2 + v^2n^2\pi^2\right)}$, $B_n = \dfrac{4L}{n\pi}\left[\dfrac{\kappa v^2L}{\omega^2L^2 + v^2n^2\pi^2} + \dfrac{(-1)^{(n-1)/2}}{n\pi}\right]$

11.185(b) $(2/n\pi)\left[\cos(n\pi/3) - \cos(2n\pi/3)\right] - (-1)^n 2/n^5$

11.187 $y(x,t) = L^2/(L^2+\pi^2)\left[1 - \cos(\sqrt{1+\pi^2/L^2}\;t)\right]\sin(\pi x/L)$

11.189 $\displaystyle\sum_{n=1}^{\infty}\left[\left(b_{fn} - \dfrac{[1-(-1)^n]2L^2}{n\pi(n^2\pi^2 - L^2)}\right)e^{-n^2\pi^2t/L^2} + \dfrac{[1-(-1)^n]2e^{-t}L^2}{n\pi(n^2\pi^2 - L^2)}\right]\sin\left(\dfrac{n\pi}{L}x\right)$ where $b_{gn} = (2/L)\int_0^L g(x)\sin(n\pi x/L)dx$

11.191 $\sum_{n=1}^{\infty} 4\kappa/(n\pi)\left(1 - e^{L^2t/(n^2\pi^2)}\right)\sin(n\pi x/L)$

11.193 $\displaystyle\sum_{n=1}^{\infty}\dfrac{4L^2}{n^3\pi^3}\left(H\dfrac{e^{n\pi y/L} - e^{-n\pi y/L}}{e^{n\pi H/L} - e^{-n\pi H/L}} - y\right)\sin\left(\dfrac{n\pi}{L}x\right)$ (odd n only)

11.195 $\displaystyle\sum_{n=1}^{\infty}\dfrac{4\kappa L^2}{L^2 + n^2\pi^2}\left(\dfrac{e^{\sqrt{1+n^2\pi^2/L^2}\;y} + e^{\sqrt{1+n^2\pi^2/L^2}\;(H-y)}}{1 + e^{\sqrt{1+n^2\pi^2/L^2}\;H}} - 1\right)\sin\left(\dfrac{n\pi}{L}x\right)$, odd n only

11.199(b) $f(x) = \sum_{n=1}^{\infty}(-1)^{n-1}(2/n\pi)\sin(n\pi x)$

11.201(b) $a''_{sn}(t) + (n^2\pi^2 c_s^2/L^2)a_{sn}(t) = -c_s^2 a_{qn}$

(c) $a_{sn}(t) = A_n \sin(n\pi c_s t/L) + B_n \cos(n\pi c_s t/L) - (a_{qn}L^2/n^2\pi^2),\ a_{s0}(t) = -(1/2)c_s^2 a_{q0}t^2 + Ct + D$

(d) $s(x,t) = -(1/2)c_s^2 a_{q0}t^2 + \sum_{n=1}^{\infty}(a_{qn}L^2/n^2\pi^2)\left[\cos(n\pi c_s t/L) - 1\right]\cos(n\pi x/L)$

11.203(c) $L^2/(14\pi^2\varepsilon_0)\sin(\pi x/L)\sin(2\pi y/L)\sin(3\pi z/L)$

11.205(b) $R_n(\rho) = J_0\left(\alpha_{0,n}\rho/a\right)$

(c) $$\frac{d^2A_{zn}}{dt^2} + \frac{v^2\alpha_{0n}^2}{a^2}A_{zn} = \frac{2}{a^2J_1^2(\alpha_{0,n})}\int_0^{a/2}\kappa\cos(\omega t)J_0\left(\frac{\alpha_{0,n}}{a}\rho\right)\rho d\rho$$

(d) $(d^2A_{zn}/dt^2) + (v^2\alpha_{0n}^2/a^2)A_{zn} = \gamma_n\kappa\cos(\omega t)$

(e) $A_{zn} = \gamma_n\kappa a^2/(v^2\alpha_{0n}^2 - a^2\omega^2)\left[\cos(\omega t) - \cos(v\alpha_{0n}t/a)\right]$

(f) $z(\rho,t) = \sum_{n=1}^{\infty}\gamma_n\kappa a^2/(v^2\alpha_{0n}^2 - a^2\omega^2)\left[\cos(\omega t) - \cos(v\alpha_{0n}t/a)\right]J_0(\alpha_{0n}\rho/a)$

11.209 $\hat{y} = \delta(p)\left(1 - e^{-t}\right) + \hat{f}e^{-c^2p^2t}$

11.211 $y(x,t) = \kappa(1 - e^{-t})$

11.213 $$\hat{y} = \frac{1}{2\sqrt{\pi}(c^2p^2 + 1 - \omega^2)}e^{-(p/2)^2}\left[\cos(\omega t) - \cos\left(\sqrt{c^2p^2+1}\,t\right)\right] + \frac{2e^{-ip/2} - e^{-ip} - 1}{2\pi p^2\sqrt{c^2p^2+1}}\sin\left(\sqrt{c^2p^2+1}\,t\right)$$

11.215 $(Qi/\pi p^3)(1 - \cos p)\left[1 - \cos(vpt)\right]$

11.217 $(Qv^2t^2\delta(p)/2) + (Gde^{-d^2p^2/4}/2vp\sqrt{\pi})\sin(vpt)$

11.219 $\hat{y} = \sin p\left(1 - e^{-(p^2-ip)t}\right)/\left[\pi p(p^2 - ip)\right]$

11.221(b) $C(p) = -e^{-(p/2)^2}/[2\sqrt{\pi}\left(p^2 + 2\pi^2\varepsilon_0\right)]$

11.223(a) $s^2U + s\sin(\pi x) - \partial^2U/\partial x^2 = s\sin(\pi x)/(1+s^2)$

(b) $U(0,s) = 0,\ U(1,s) = 1/s^2$

(c) $$U(x,s) = -\frac{s^3}{(s^2+1)(s^2+\pi^2)}\sin(\pi x) + \frac{e^{sx} - e^{-sx}}{s^2(e^s - e^{-s})}$$

11.225 $$Y(x,s) = \frac{1}{s^2(1+s^2)}\left[1 - \frac{e^{(s/c)x} + e^{s/c}e^{-(s/c)x}}{\left(1 + e^{s/c}\right)}\right]$$

11.227 $Y(x,s) = (1/s^2)\sinh[\sqrt{s}(1-x)]/\sinh(\sqrt{s})$

11.229 $$Y(x,s) = \frac{x}{s^3} + \frac{e^{x\sqrt{s}/c} - e^{-x\sqrt{s}/c}}{s^3\left(e^{-\sqrt{s}/c} - e^{\sqrt{s}/c}\right)} = \frac{x}{s^3} - \frac{1}{s^3}\frac{\sinh(x\sqrt{s}/c)}{\sinh(\sqrt{s}/c)}$$

11.231(b) $(\partial^2Y/\partial x^2) = (s^2/v^2)Y$

(e) $Y(x,s) = [y_0\omega/(\omega^2 + s^2)]e^{-(s/v)x}$

11.233(a) $z(x,0) = 2\sin(\pi x/6)$

(c) $\sin(5\pi x/6)\,e^{(-25\pi^2+20)t/16}$

(e) $\sin(\pi x/6)\,e^{(-\pi^2+20)t/16} + \sin(5\pi x/6)\,e^{(-25\pi^2+20)t/16}$

11.235(a) $\sum_{n=1}^{\infty}(2s/n\pi)[\cos(n\pi/3) - \cos(2n\pi/3)]\sin(nx)$

(b) $\sum_{n=1}^{\infty}(2s/n^2v\pi)[\cos(n\pi/3) - \cos(2n\pi/3)]\sin(nx)\sin(nvt)$

11.241 $y(x,t) = \frac{3}{2}x - \frac{1}{2}x^2 + \sum_{n=1}^{\infty}\left[-\frac{18(1-(-1)^n)}{n^3\pi^3} + \frac{2}{n\pi}\left(\cos\left(\frac{n\pi}{3}\right) - \cos\left(\frac{2n\pi}{3}\right)\right)\right]$
$\sin\left(\frac{n\pi}{3}x\right)e^{-\frac{n^2\pi^2}{9}t}$

11.243 $y(x,t) = \frac{\alpha^2 x_0}{2\sqrt{\pi}\beta}\int_{-\infty}^{\infty}\frac{1}{c^2p^2-\omega^2}e^{-p^2x_0^2/(4\beta^2)}\left(\cos(\omega t) - \cos(cpt)\right)e^{ipx}dp$

11.245 $y(x,t) = \sum_{n=1}^{\infty} -108(1+2(-1)^n)/(\pi^3n^3)\sin(n\pi x/3)e^{-n^2\pi^2 t}$

11.247 The solution $u(x,t)$ is the inverse Laplace transform of
$\frac{1}{s(1+s)}\left[1 + e^{(x-1)/2}\left(\frac{e^{1/2} - e^{(1/2)\sqrt{1+4s}}}{e^{\sqrt{1+4s}}-1}e^{(1/2)\sqrt{1+4s}x} + \frac{e^{1/2} - e^{-(1/2)\sqrt{1+4s}}}{e^{-\sqrt{1+4s}}-1}e^{-(1/2)\sqrt{1+4s}x}\right)\right]$

11.249 $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} 16V/[mn\pi^2\sin\left(\pi\sqrt{n^2+m^2}\right)]\sin(m\pi x/L)\sin\left(n\pi y/L)\sin((\pi/L)\sqrt{m^2+n^2}\,z\right)$
for odd m and n

11.251 $y(x,t) = 1/(81\pi^4)\left(e^{-9\pi^2 t} + 9\pi^2 t - 1\right)\sin(3\pi x)$

11.253 $y(x,t) = \sum_{n=1}^{\infty}\left(B_n/A_n + C_n e^{-A_n t}\right)\sin(n\pi x/3)$ where $A_n = \alpha^2n^2\pi^2/9 + \beta^2$,
$B_n = 6/(n^2\pi^2)\sin(n\pi/3) + (6-2n\pi)/(n^2\pi^2)\sin(2n\pi/3) + 2/(n\pi)\cos(2n\pi/3)$,
$C_n = -18/(\pi^3n^3)\left[n\pi\sin(n\pi) + 2\cos(n\pi) - 2\right] - B_n/A_n$

11.255 $x - \sum_{n=1}^{\infty} 8/(n^3\pi^3)\sin(n\pi x)e^{-n^2\pi^2 t}$ for odd n only

11.257 $\hat{y}(p) = e^{-(p/2)^2}/[2\sqrt{\pi}p^2(v^2-1)]\left[e^{ipt} - (i/v)\sin(vpt) - \cos(vpt)\right]$

11.259(a) $\partial^2V/\partial r^2 + \partial^2V/\partial\phi^2 = e^{-r}\sin\phi$

(b) $r \geq 0$

(d) $\hat{V}_s = -2p\sin\phi/\left[\pi(1+p^2)^2\right] + A(p)e^{p\phi} + B(p)e^{-p\phi}$

(e) $A(p) = B(p) = 0$

Chapter 12

12.1 $\sum_{n=1}^{57} n$

12.3 $\prod_{n=1}^{100} n$

12.5 $\sum_{n=0}^{28} (-1)^n \sin[(2n+1)x]$

12.7 $\sum_{n=0}^{28} (4n^2 + 2n + 1)$

12.9(a) $a_n = a_{n-1} + 3$

(b) $a_n = 14 + 3(n-1)$, $a_{100} = 311$

(c) $\sum_{n=1}^{\infty} [14 + 3(n-1)]$

12.11(b) $f(n) = \ln(n-3)$

(c) $a = 3$, $b = 8$

12.13 $(n+1)c_{n+1}$

12.15 $1 + 3x + \sum_{n=2}^{\infty} 6x^n$

12.17 $\sum_{n=0}^{\infty} \left(n^2 + 3n + 4\right) x^n$

12.19(a) $\sum_{n=0}^{\infty} c_n(n+3)x^{n-2} - \sum_{n=0}^{\infty} c_n(n+3)x^n$

(b) $\sum_{n=-2}^{\infty} c_{n+2}(n+5)x^n - \sum_{n=0}^{\infty} c_n(n+3)x^n + \sum_{n=-1}^{\infty} c_{n+1}(n-6)x^n + \sum_{n=0}^{\infty} c_n x^n$

(c) $3c_0x^{-2} + 4c_1x^{-1} - 7c_0x^{-1} + \sum_{n=0}^{\infty} \left[c_{n+2}(n+5) + c_{n+1}(n-6) - c_n(n+2)\right] x^n = 8x^{-1}$

(d) $c_0 = 0$

(e) $c_1 = 2$

(f) $c_{n+2} = \left[c_n(n+2) - c_{n+1}(n-6)\right]/(n+5)$

(g) $c_2 = 12/5$

(h) $c_3 = 3$, $c_4 = 108/35$

12.21(a) even

(b) odd

(c) ∞, ∞

(d) $\infty, -\infty$

(e) $1, 0$

(g) $1, -1$

12.23(b) $A = (x_0 + v_0/\omega)/2, B = (x_0 - v_0/\omega)/2$

(d) $C = v_0/\omega, D = 0$

(e) $C = 0, D = x_0$

(f) $C = v_0/\omega, D = x_0$

12.25 $\left(\sqrt{\pi}/2\right)(\text{erf } 7 - \text{erf } 5)$

12.27(a) 0

(b) always increasing

(c) always concave down

12.29 $\text{erf } x = \left(2/\sqrt{\pi}\right)x - 2x^3/\left(3\sqrt{\pi}\right) + \ldots$

12.31(a) $\text{erf}\left[x/(s\sqrt{2})\right]$

(b) 1

(d) $16/\sqrt{2}$

12.33 2

12.35 139

12.37 $\Gamma(x+21)/\Gamma(x+4)$

12.43(a) $S = k_B\left[\left(q+N-\frac{1}{2}\right)\ln(q+N-1) - \frac{1}{2}\ln(2\pi) - \left(q+\frac{1}{2}\right)\ln q - \left(N-\frac{1}{2}\right)\ln(N-1)\right]$

(b) $\dfrac{dS}{dq} = k_B\left(\dfrac{1-N}{2q(q+N-1)} + \ln(q+N-1) - \ln q\right)$

(c) $\dfrac{dS}{dq} = k_B\left(\dfrac{-N}{2q(q+N)} + \ln(q+N) - \ln q\right)$

(d) **(i)** $\dfrac{dS}{dq} = k_B\left[\dfrac{-Nx^2}{2(1+Nx)} + \ln(1+Nx)\right]$

(ii) $dS/dq \approx k_B N x$

(iii) $dS/dq \approx k_B N/q$

(e) $dS/dU = k_B N/(q\epsilon)$

(f) $T = U/(k_B N)$

12.49(g) 1

12.53(a) $c_2 = (1/2 - c_0)/2, c_3 = (-1/4 - c_1)/6, c_4 = (1/8 + c_1 - c_2)/12$

(b) $y(x) = x + (1/4)x^2 - (5/24)x^3 + (7/96)x^4$

12.55 $y = -2 - 2x - x^2$

12.57 $y(x) = 2 + x - x^2 - x^3/3 + x^4/4$

12.59 $x(t) = 1 + t^2/2 + t^4/16$

12.61 $y(x) = 24 + 24x + 12x^2 - 4x^3 - x^4$

12.65 $H_{\text{even}}(x) = c_0\left[1 - \dfrac{2k}{2!}x^2 + \dfrac{2^2k(k-2)}{4!}x^4 - \dfrac{2^3k(k-2)(k-4)}{6!}x^6 + \ldots\right]$

12.69 $(3/5)P_1(x) + (2/5)P_3(x)$

12.71 $(9/5)P_1(x) - (84/55)P_3(x)$

12.73 $(1/2)P_0(x) - (5/8)P_2(x)$

12.75 $(\sin 1)P_0(x) + 5(3\cos 1 - 2\sin 1)P_2(x)$

12.81(b) $\Theta = AP_l(\cos\theta) + BQ_l(\cos\theta)$ where $l(l+1) = k$

12.83(a) $A_n = \dfrac{2n+1}{2R^n}\displaystyle\int_0^\pi \left(\cos^2\theta\right) P_n(\cos\theta)\sin\theta\, d\theta$

(b) $A_n = \dfrac{2n+1}{2R^n}\displaystyle\int_{-1}^1 u^2 P_n(u)\, du$

(c) $A_0 = 1/3$, $A_2 = 2/(3R^2)$, and $A_1 = A_3 = 0$

12.85(b) $\displaystyle\sum_{j=0}^{l/2}(-1)^{l/2-j}\frac{l!}{(l/2-j)!(l/2+j)!}x^{l+2j}$

(c) $(-1)^{l/2}\dfrac{l!}{[(l/2)!]^2}x^l$, $(-1)^{l/2-1}\dfrac{l!}{(l/2-1)!(l/2+1)!}x^{l+2}$, $(-1)^{l/2}\dfrac{l!}{(l/2-2)!(l/2+2)!}x^{l+4}$

(d) $(-1)^{l/2}\left[\dfrac{l!}{(l/2)!}\right]^2 + (-1)^{l/2-1}\dfrac{l!}{(l/2-1)!(l/2+1)!}\dfrac{(l+2)!}{2!}x^2$
$+(-1)^{l/2}\dfrac{l!}{(l/2-2)!(l/2+2)!}\dfrac{(l+4)!}{4!}x^4$

(e) $(-1)^{l/2}\left[\dfrac{l!}{(l/2)!}\right]^2\left[(-1)\dfrac{l(l+1)}{2!}x^2 + \dfrac{l(l+1)(l-2)(l+3)}{4!}x^4\right]$

12.87(c) $(-1)^l\dfrac{1}{2^{2l}(l!)^2}\displaystyle\int_{-1}^1 (x^2-1)^l\frac{d^{2l}}{dx^{2l}}\left[(x^2-1)^l\right]dx$

(d) $(2l)!$

12.89(a) $y_{1/2}$

(b) $3 - 6x + x^2$

12.91 $c_0 + d_0x^{3/2}\left(1 - 3x/20 + 3x^2/224\right)$

12.93 $c_0x^{1/5}\left(1 - 5x^2/6\right) + d_0x^{-1/5}\left(1 - 5x^2/4\right)$

12.95(d) $y(x) = Ax^{2/3} + Bx^{-2}$

12.97(e) $c_0(1/x - 1/2)$

12.99 $y = Ax^i + Bx^{-i}$

12.101(b) $\displaystyle\sum_{n=0}^\infty c_n(r+n-1)(r+n)x^{r+n} + \sum_{n=0}^\infty c_n(r+n)x^{r+n} + \sum_{n=0}^\infty c_nx^{r+n+2} - \sum_{n=0}^\infty p^2c_nx^{r+n} = 0$

(c) $c_0(r^2-p^2)x^r + c_1\left[(r+1)^2-p^2\right]x^{r+1} + \displaystyle\sum_{n=2}^\infty\left(c_n\left[(r+n)^2-p^2\right] + c_{n-2}\right)x^{r+n} = 0$

(d) $r = \pm p$

(e) $c_1 = 0$

(f) $c_n = c_{n-2}/\left[p^2 - (r+n)^2\right]$

12.103(a) $c_{2m} = c_{2m-2}/\left[4m(p-m)\right]$

(b) $c_6 = c_0/\left[4^3(3!)(p-1)(p-2)(p-3)\right]$

(c) $c_{2m} = c_0/\left[4^m(m!)(p-1)(p-2)(p-3)\ldots(p-m)\right]$

(d) $c_{2m} = c_0(-1)^m\Gamma(-p+1)/\left[4^m(m!)\Gamma(p-m+1)\right]$

12.107(b) $H_p^{(1)} = \left[1 + i\cot(\pi p)\right]J_p(x) - i\csc(\pi p)J_{-p}(x)$, $H_p^{(2)} = \left[1 - i\cot(\pi p)\right]J_p(x) + i\csc(\pi p)J_{-p}(x)$

12.111 $y = AJ_0[(1-i)x/\sqrt{2}] + BY_0[(1-i)x/\sqrt{2}]$

12.113 $y(x) = \sqrt{x}\left[AJ_{n+1/2}(x) + BY_{n+1/2}(x)\right]$

12.115(a) $-1 < x < 1$

(b) $(1-x^2)y''(x) - 2xy'(x) + \lambda y(x) = 0$

(c) $AP_l(x) + BQ_l(x)$ where $\lambda = l(l+1)$

(e) $\lambda = l(l+1)$ for non-negative integer l, $y(x) = P_l(x)$

(f) $\pm 1/\sqrt{3}$

12.117(a) x

(b) $xf(x)J_0(\alpha_{0,m}x) = \sum_{n=1}^{\infty} c_n x J_0(\alpha_{0,n}x)J_0(\alpha_{0,m}x)dx$

(c) $\int_0^1 xf(x)J_0(\alpha_{0,m}x)dx = (1/2)c_m J_1^2(\alpha_{0,m})$

(d) $c_m = 2\left(\int_0^1 xf(x)J_0(\alpha_{0,m}x)dx\right)/J_1^2(\alpha_{0,m})$

12.119 $c_m = 2\left(\int_0^1 xf(x)J_1(\alpha_{1,m}x)dx\right)/J_2^2(\alpha_{1,m})$

12.121 $c_m = \left(\int_{-\infty}^{\infty} e^{-x^2}f(x)H_m(x)dx\right)/\left(\sqrt{\pi}\,2^m m!\right)$

12.123 $\pi/2 - \sum_{n=-\infty}^{\infty} 2e^{inx}/(n^2\pi)$ (odd n only)

12.133(a) $y_m''(x) + \lambda_m y_m(x) = 0$

(b) $y_n''(x) + \lambda_n y_n(x) = 0$

(c) $y_m''(x)y_n(x) - y_n''(x)y_m(x) + y_m(x)y_n(x)\left(\lambda_m - \lambda_n\right) = 0$

(d) $\left[y_n(x)y_m'(x) - y_m(x)y_n'(x)\right]_0^L + \left(\lambda_m - \lambda_n\right)\int_0^L y_m(x)y_n(x)dx = 0$

12.141(a) 1

12.147(a) $\hat{B}\hat{A}\psi_m + \lambda_m\psi_m = 0$

(b) $\lambda_n = \lambda_m - 5$

(c) $\hat{B}\psi_0 = 0$

(d) $\lambda = 5, 10, 15, \ldots$

12.151 $\sqrt[4]{m\omega/\hbar\pi}$

12.153(a) $\hbar\hat{L}_+, -\hbar\hat{L}_-$

(b) $c_m + \hbar$ and $c_m - \hbar$

12.155 $y = c_0 + c_1 t - c_0 t^2/2 + (1 - c_0 - c_1)t^3/6 + (c_0 - 2c_1)t^4/24 + \ldots$

12.157 $y(t) = c_0 + c_1 t - (c_0 + c_1)t^2/2 + (c_0 + 1)t^3/6 + (c_1 - 1)t^4/24 + \ldots$

12.159 $y(t) = c_0 t^{-1/2} + d_0 - (11/16)c_0 t^{3/2} - (1/5)d_0 t^2 + \ldots$

12.161(e) $L_0(x) = 1$

(f) $L_1(x) = 1 - x$

(g) $L_2(x) = x^2/2 - 2x + 1$, $L_3(x) = -x^3/6 + 3x^2/2 - 3x + 1$

12.163(f) $r = r_0 + v_0 t - \dfrac{GM}{2r_0^2}t^2 + \dfrac{GMv_0}{3r_0^3}t^3 + \ldots$

Chapter 13

13.1(a) $u = x^2 - y^2$, $v = 2xy$

(b) $|f| = |z|^2$, $\phi_f = 2\phi_z$

13.5 $(1/2)\sin x\,(e^{-y} + e^{y}) - i(1/2)\cos x\,(e^{-y} - e^{y})$

13.11(a) $\phi_{\sqrt{z}} = (1/2)\phi_z$, $|\sqrt{z}| = \sqrt{|z|}$

(b) $2e^{i\pi/4}$, $2e^{-3i\pi/4}$.

(c) $\pm\pi$

13.13 Zeros: $z = -1$ and $z = 2i$. Poles: $z = 3$ and $z = -5 + 2i$.

13.15(a) $z = -1 - i$

(b) $z = -4i$

13.17 nz_0^{n-1}

13.21 analytic

13.23 analytic

13.27 $V = V_0 \ln\left(\sqrt{x^2 + y^2}/R\right)/(\ln 3)$

13.29 $V(x, y) = (1/2)(V_2 - V_1)y + (5/2)V_1 - (3/2)V_2$

13.33(a) vertical lines

(b) The boundary conditions would only depend on x.

13.35(a) $\sqrt{\dfrac{\sqrt{x^2 + y^2} + x}{2}} + i\sqrt{\dfrac{\sqrt{x^2 + y^2} - x}{2}}$

(b) parabola

13.37(b) First-order pole at $z = 0$, second-order pole at $z = -1$

(c) $z = -1$

(d) $-2e^{-1}$

(e) $-4\pi i e^{-1}$

(f) 0

(g) $2\pi i(1 - 2/e)$

13.39 0

13.41 $2\pi i$

13.43 0

13.45 $(2/3)\pi \ln(3i)$

13.47 0

13.49 $(\pi i/120)(e^{-1} + e)$

13.53(a) $\displaystyle\sum_{n=1}^{2\pi/(\Delta\phi)} \left(e^{i\Delta\phi} - 1\right)$

(b) $2\pi i$

13.55(a) R^{m-n}

(b) πR

13.57(b) 0

13.59 They both give 0.

13.63 $\pi/3$

13.65 $-5\pi/6$

13.67 π/e^5

13.69 $\pi/(4e^4)$

13.71(a) $\int_0^{2\pi} dx/\left[5-2i(e^{ix}-e^{-ix})\right]$

(b) $\int_0^{2\pi} du/(2u^2+5iu-2)$

(c) $(2u+i)(u+2i)$

(d) unit circle

(e) $2\pi/3$

13.73 $\pi/6$

13.75 $2\pi/\sqrt{3}$

13.77 $f(t)=e^t$

13.79 $f(t)=1$

13.81 $f(t)=e^{-3t}(1-3t+t^2)$

13.87 $\sqrt{2}\,\pi/\sqrt[4]{3}$

13.89 $\pi/\left[|x_0|^k\sin(\pi k)\right]$

13.91(b) $(\ln|z|)^2-\phi^2+2\ln|z|\phi i$

(c) $4\pi^2x_0-4\pi iR$

(d) 0

(f) $4\pi x_0\left(-\pi+i\ln x_0-i\right)$

(g) $x_0\ln x_0-x_0$

13.93(b) $1+1/z+1/(2z^2)+1/(6z^3)+\ldots+1/(n!z^n)+\ldots$

(c) Essential singularity

(d) 1

(e) $2\pi i$

13.95(a) 1

(b) $\sqrt{2}$

(c) π

13.97(a) $1/\sqrt{2}\left[(z-\pi/4)^{-1}+1-(1/2)(z-\pi/4)+\ldots\right]$

(b) $1/\sqrt{2}$

(c) $\sqrt{2}\,\pi i$

13.99 $f(z)=-z^{-1}-z-z^3+\ldots,\ R=1$

13.101(a) $a=-1,\ b=1$

(b) $f(z)=-z^{-1}-1+z-z^2+\ldots$

(d) $f(z)=\sum_{n=2}^{\infty}z^{-n}$

13.103 $g^{(n-1)}(z_0)/(n-1)!$

13.111 the disk enclosed by the unit circle.

13.113 quarter-circle between $e^{-i\pi/4}$ and $e^{i\pi/4}$

13.115 fourth quadrant between the circles of radius $1/2$ and 1

13.117 circle of radius $7/26$ centered on $(29/26)+i(7/13)$

13.121(a) $|f|^2 = (x^2+y^2)^2 + 2x^3 + 2xy^2 + x^2 + y^2$

(c) $T = 0$ on the inner circle and $T = 1$ on the outer.

(d) $\left(\ln\sqrt{u^2+v^2}\right)/(\ln 2)$

(e) $u(x,y) = x^2 + x - y^2$, $v(x,y) = 2xy + y$

(f) $\left(\ln\sqrt{(x^2+y^2)^2 + 2x^3 + 2xy^2 + x^2 + y^2}\right)/(\ln 2)$

(h) **(i)** $\ln(36/25)/\ln(2) \approx 0.53$

(ii) $(36/25, 0)$

(iii) $\ln(36/25)/\ln(2) \approx 0.53$

(iv) La-de-da

13.123(a) $V(u,v) = \left(\ln\sqrt{u^2+v^2}\right)/(\ln 2)$

(b) $z = \sqrt{(1/2)e^{i\phi}+1}$ and $z = \sqrt{e^{i\phi}+1}$ for $0 \le \phi \le 2\pi$

(c) $f(z) = z^2 - 1$

(d) $u(x,y) = x^2 - y^2 - 1$ and $v(x,y) = 2xy$

(e) Find the potential function in the region bounded by $z = \sqrt{(1/2)e^{i\phi}+1}$ (held at $V = -1$) and $z = \sqrt{e^{i\phi}+1}$ (held at $V = 0$) for $0 \le \phi \le 2\pi$ by using the mapping $f(z) = z^2 - 1$.

(f) $V(x,y) = \left(\ln\sqrt{(x^2+y^2)^2 + 2(y^2-x^2) + 1}\right)/(\ln 2)$

13.125 $T(x,y) = \dfrac{1}{\ln 2}\ln\left(2\sqrt{\dfrac{1-6x}{x^2+y^2}+9}\right)$

13.127 $T(x,y) = \dfrac{2}{\pi}\tan^{-1}\left(\dfrac{(x^2+y^2)^{1/4}\cos\left(\left[\tan^{-1}(y/x)\right]/2\right)}{(x^2+y^2)^{1/4}\sin\left(\left[\tan^{-1}(y/x)\right]/2\right)-1}\right)$

13.129(b) Radius $1/3$ centered at $z = -2/3$

(c) $Re(f(z)) = -1/4$

(d) $V = A\ln\left(B\sqrt{\dfrac{4x+1}{x^2+y^2}+4}\right)$ where $A = \dfrac{V_L - V_C}{\ln 2}$ and $B = e^{\left(\frac{V_C \ln 2}{V_L - V_C}\right)}$

13.131 $T = (2/\pi)\tan^{-1}(\cot x \tanh y)$

13.133(a) $V = v/\pi$

13.135(a) $\psi = \ln(x^2+y^2)$

(b) $\psi = y[1 - 1/(x^2+y^2)] + \ln(x^2+y^2)$

13.137(a) $\psi(x,y) = \left(3x^2y - y^3\right)\left(1 - R^2/(x^2+y^2)^3\right)$

(b) $\vec{v} = \left[3(x^2-y^2) - \dfrac{3R^2}{(x^2+y^2)^4}(x^4 - 6x^2y^2 + y^4)\right]\hat{\imath} - 6xy\left[1 + 2R^2\dfrac{x^2-y^2}{(x^2+y^2)^4}\right]\hat{\jmath}$

13.141(b) $\chi(z) = \psi(z) + \chi(z_0)$

13.143(a) $\psi = k\tan^{-1}(y/x)$

(b) $\vec{v} = \dfrac{kx}{x^2+y^2}\hat{\imath} + \dfrac{ky}{x^2+y^2}\hat{\jmath}$

(c) $k = 800\ \text{cm}^2/\text{s}$

13.145(a) Maps the upper half-plane to itself

(b) first quadrant

(c) segment from $(0,0)$ to $\left(\sqrt{2}, 0\right)$.

(d) $(x^2+y^2)^2 = 2(x^2-y^2)$

13.149(a) $z = x + i$ where $x \leq 0$

(b) $z = x - i$ where $x \leq 0$

13.151(f) The branch cut is on the imaginary axis between $z = -i$ and $z = i$.

13.153 $\pi \sin(2\pi k) / \left[x_0^k - x_0^k \cos(2\pi k)\right]$

13.155(a) $T(u, v) = (2T_0/\pi) \tan^{-1}(v/u)$

(c) $T(x, y) = \dfrac{2T_0}{\pi} \tan^{-1}\left(\dfrac{\tan^{-1}(y/x)}{\ln\sqrt{x^2 + y^2}}\right)$

13.157(a) $u(x, y) = (1/2)\sin x(e^y + e^{-y})$, $v(x, y) = (1/2)\cos x(e^y - e^{-y})$

(c) $f(x + 5i) = 74.21 \sin x + 74.20i \cos x$ where $0 \leq x \leq \pi/2$

(a) $T(u, v) = (2/\pi)\tan^{-1}(v/u)$

(b) $T(x, y) = \dfrac{2}{\pi} \tan^{-1}\left(\dfrac{\cos x(e^y - e^{-y})}{\sin x(e^y + e^{-y})}\right)$