Motivating Exercise for Fourier Series and Transforms: Discovering Extrasolar Planets

We typically imagine the Earth orbiting a stationary sun, but that is only an approximation. While the sun pulls the Earth around, the Earth pulls the sun around too, so they orbit together around their mutual center of mass. The period of this rotation is of course 365 days, so its angular frequency is $\omega = 2\pi/365 \text{ day}^{-1}$.

Astronomers therefore look for small, regular oscillations in other stars as a way of determining if they have their own planets. Using this technique, Mayor and Queloz in 1995 announced what is now considered the first definitive detection of a planet orbiting a star other than our sun.[1]

In a simpler world we would see a sine wave as the star moves back and forth, and the period of that sine wave would be the orbital period of the planet. But real data is messier because stars can have more than one planet, and because effects such as atmospheric distortion create "noise" in the data.

Suppose you had been measuring a star carefully for 14 years and collected the data shown in Figure 1. Does this star have planets going around it? How many and with what periods? Or did your five-year-old just scribble random dots on your page of data? It's virtually impossible to tell. You need to know if there are regular sinusoidal variations hidden within the random ups-and-downs of those data points.

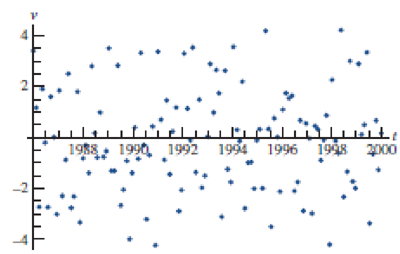

Figure 1: Velocity of a star vs. time

"Fourier analysis" is designed to answer that question. It allows you to take a function of time and figure out how much it varies with different frequencies. Figure 2 shows the "Fourier transform" $\hat{v}(\omega)$ of the data points $v(t)$ shown in Figure 1.

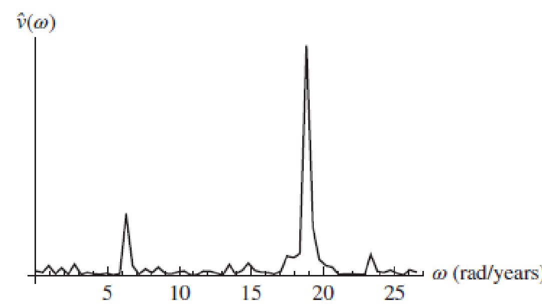

Figure 2: The "Fourier transform" shows how much variation $v(t)$ has at different frequencies.

It is vital to understand that Figure 2 does not represent any new data; it is calculated by analyzing the data in Figure 1. You will learn how to perform this analysis using a "discrete Fourier transform." Right now we are handing you Figure 2, but you can still interpret it.

[1]M. Mayor, D. Queloz (1995). "A Jupiter-mass companion to a solar-type star". Nature 378 (6555): 355-359. Other claims of "exoplanets" had been claimed earlier, but were considered more controversial. In 1992 Wolszczan and Frail announced the detection of two planets orbiting a pulsar, but Mayor and Queloz's discovery was the first clear planet orbiting a regular star.

1. The Fourier transform shows a sharp spike at approximately $\omega = 19$. This indicates that your star's velocity has a high-amplitude oscillation with angular frequency 19. The only likely explanation is a planet orbiting the star. What is the period of that planet's rotation? (That is, how long is a "year" on that planet?)

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2. The Fourier transform also shows a spike at roughly $\omega = 6$, indicating a second planet. How long is a year on that planet?

3. Which planet do you think has the largest mass (and hence the biggest effect on the star's velocity)? Explain how you got your answer from Figure 2.