Discovery Exercise for Eigenvectors and Eigenvalues

Matrix **S** is a point matrix: $\mathbf{S} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$.

1. Draw the shape that matrix **S** describes.

Matrix **B** represents a transformation: $\mathbf{B} = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}$.

2. Find the matrix **BS** and draw the resulting shape.

3. Describe in words: what effect does matrix **B** have on this (or any other) shape?

See Check Yourself #43 at felderbooks.com/checkyourself

Matrix **T** represents a different transformation: $\mathbf{T} = \begin{pmatrix} 3 & 1 \\ 3 & 5 \end{pmatrix}$.

4. Find the matrix **TS** and draw the resulting shape.

It's not at all clear what $\mathbf{T}$ does, is it? It seems to move everything around almost randomly! But let's try it on a new shape: $\mathbf{C} = \begin{pmatrix} 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 4 & 3 & 0 \end{pmatrix}$.

5. Find the matrix $\mathbf{TC}$.

6. Draw the shapes described by matrix $\mathbf{C}$ and matrix $\mathbf{TC}$.

7. For this particular shape, it should be clear what effect $\mathbf{T}$ had—and in fact, it has the same effect on any shape. (If you sketch this too casually it might not be clear, but if you graph it carefully on graph paper or a computer you should be able to see it.) Fill in the blanks in the following sentence:

   Matrix $\mathbf{T}$ stretches a shape by a factor of ____ in the direction __________, and it also stretches a shape by a factor of ____ in the direction __________.