Equations for LATTICEEASY models

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Contents

1 Introduction

This document lists the equations needed to enter these models into the LATTICEEASY program. The equations are explained in the section "Writing Down the Equations for a New Model" in the LATTICEEASY documentation. Program variables (denoted with a subscript pr) are defined as

$$f_{pr} \equiv A a^r f;\ \vec{x}_{pr} \equiv B\vec{x};\ dt_{pr} \equiv B a^s dt. \tag{1}$$

For a model dominated by a potential of the form

$$V = \frac{cpl}{\beta}\phi^\beta \tag{2}$$

the default rescalings are

$$A = \frac{1}{\phi_0};\ B = \sqrt{cpl}\,\phi_0^{-1+\beta/2};\ r = \frac{6}{2+\beta};\ s = 3\frac{2-\beta}{2+\beta} \tag{3}$$

where ϕ_0 is the initial value of the field ϕ. The rescaled program potential is defined to be

$$V_{pr} \equiv \frac{A^2}{B^2} a^{-2s+2r} V \tag{4}$$

and the effective mass (in program units) is

$$m_{f,pr}^2 = a^{2s+2}\frac{\partial^2 V_{pr}}{\partial f_{pr}^2}. \tag{5}$$

2 TWOFLDLAMBDA

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2. \tag{6}$$

The dominant potential term is $\frac{1}{4}\lambda\phi^4$ so

$$\beta = 4;\ cpl = \lambda \tag{7}$$

$$A = \frac{1}{\phi_0};\ B = \sqrt{\lambda}\phi_0;\ r = 1;\ s = -1. \tag{8}$$

$$f_{pr} = \frac{a}{\phi_0}f;\ \vec{x}_{pr} = \sqrt{\lambda}\phi_0\vec{x};\ dt_{pr} = \sqrt{\lambda}\phi_0\frac{dt}{a} \tag{9}$$

The potential in terms of program variables is

$$V = \left(\frac{\phi_0}{a}\right)^4\left(\frac{1}{4}\lambda\phi_{pr}^4 + \frac{1}{2}g^2\phi_{pr}^2\chi_{pr}^2\right) \tag{10}$$

Defining a new variable $gl \equiv \frac{g^2}{\lambda}$, the program equations are

$$V_{pr} = \frac{a^4}{\lambda\phi_0^4}V = \frac{1}{4}\phi_{pr}^4 + \frac{1}{2}gl\ \phi_{pr}^2\chi_{pr}^2 \tag{11}$$

$$\frac{\partial V_{pr}}{\partial\phi_{pr}} = \left(\phi_{pr}^2 + gl\ \chi_{pr}^2\right)\phi_{pr} \tag{12}$$

$$\frac{\partial V_{pr}}{\partial\chi_{pr}} = gl\ \phi_{pr}^2\chi_{pr} \tag{13}$$

$$\frac{\partial^2 V_{pr}}{\partial\phi_{pr}^2} = 3\phi_{pr}^2 + gl\ \chi_{pr}^2 \tag{14}$$

$$\frac{\partial^2 V_{pr}}{\partial\chi_{pr}^2} = gl\ \phi_{pr}^2. \tag{15}$$

Finally it remains to find an appropriate value for ϕ_0. We choose to set $\frac{\partial}{\partial t_{pr}}\phi_{pr} = 0$ at the beginning of the run, so we must find the value of ϕ for which this derivative vanishes.

$$\frac{\partial}{\partial t_{pr}}\phi_{pr} \propto \left(\dot{\phi} + \frac{\dot{a}}{a}\phi\right). \tag{16}$$

During inflation we can neglect χ and consider a homogeneous field ϕ that obeys the equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \lambda\phi^3 = 0. \tag{17}$$

Using the Friedmann equation

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3}\rho} \approx \sqrt{\frac{4\pi}{3}\left(\dot{\phi}^2 + \frac{1}{2}\lambda\phi^4\right)} \tag{18}$$

and the rescaling $t' \equiv \sqrt{\lambda}t$ equation 17 becomes

$$\ddot{\phi} + \sqrt{12\pi\left(\dot{\phi}^2 + \frac{1}{2}\phi^4\right)}\dot{\phi} + \phi^3 = 0 \tag{19}$$

where dots now mean derivatives with respect to t'. This equation can be solved numerically starting from $\phi \gtrsim 1$ and $\dot{\phi} = 0$ (because of slow roll during inflation. The point at which the combination $\left(\dot{\phi} + \frac{\dot{a}}{a}\phi\right)$ vanishes is $\phi \approx .342$, so we take this value for ϕ_0.

3 ONEFLDLAMBDA

$$V = \frac{1}{4}\lambda\phi^4. \tag{20}$$

This model is just the TWOFLDLAMBDA model without the field χ. All rescalings, including the value of ϕ_0, are the same as for TWOFLDLAMBDA. The program equations are

$$V_{pr} = \frac{a^4}{\lambda\phi_0^4}V = \frac{1}{4}\phi_{pr}^4 \tag{21}$$

$$\frac{\partial V_{pr}}{\partial\phi_{pr}} = \phi_{pr}^3 \tag{22}$$

$$\frac{\partial^2 V_{pr}}{\partial\phi_{pr}^2} = 3\phi_{pr}^2 \tag{23}$$

4 NFLDLAMBDA

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}h_i^2\chi^2\sigma_i^2. \tag{24}$$

Repeated indices are summed, so this model can include an arbitrary number of fields σ_i. This model is a generalization of the TWOFLDLAMBDA model. In fact TWOFLDLAMBDA is only included as a separate model because it provides a simple illustration of a model file. Running this model with $nflds = 2$ gives the exact same results. The dominant potential term is still $\frac{1}{4}\lambda\phi^4$ and all the rescalings (including the value of ϕ_0) are the same as for TWOFLDLAMBDA.

Defining new variables $gl \equiv \frac{g^2}{\lambda}$, $hl_i \equiv \frac{h^2}{\lambda}$, the program equations are

$$V_{pr} = \frac{a^4}{\lambda\phi_0^4}V = \frac{1}{4}\phi_{pr}^4 + \frac{1}{2}gl\ \phi_{pr}^2\chi_{pr}^2 + \frac{1}{2}hl_i\ \chi_{pr}^2\sigma_{i,pr}^2 \tag{25}$$

$$\frac{\partial V_{pr}}{\partial\phi_{pr}} = \left(\phi_{pr}^2 + gl\ \chi_{pr}^2\right)\phi_{pr} \tag{26}$$

$$\frac{\partial V_{pr}}{\partial\chi_{pr}} = \left(gl\ \phi_{pr}^2 + hl_i\ \sigma_{i,pr}^2\right)\chi_{pr} \tag{27}$$

$$\frac{\partial V_{pr}}{\partial\sigma_{i,pr}} = hl_i\ \chi_{pr}^2\sigma_{i,pr} \tag{28}$$

$$\frac{\partial^2 V_{pr}}{\partial\phi_{pr}^2} = 3\phi_{pr}^2 + gl\ \chi_{pr}^2 \tag{29}$$

$$\frac{\partial^2 V_{pr}}{\partial\chi_{pr}^2} = gl\ \phi_{pr}^2 + hl_i\ \sigma_{i,pr}^2 \tag{30}$$

$$\frac{\partial^2 V_{pr}}{\partial\sigma_{i,pr}^2} = hl_i\ \chi_{pr}^2. \tag{31}$$

5 TWOFLDLAMBDACHI4

$$V = \frac{1}{4}\lambda_\phi \phi^4 + \frac{1}{2} g^2 \phi^2 \chi^2 + \frac{1}{4}\lambda_\chi^2 \chi^4 \tag{32}$$

This is the TWOFLDLAMBDA model with a χ^4 term added. The dominant potential term is $\frac{1}{4}\lambda_\phi \phi^4$ and all the rescalings (including the value of ϕ_0) are the same as for TWOFLDLAMBDA.

Defining new variables $gl \equiv \frac{g^2}{\lambda_\phi}, lcp \equiv \frac{\lambda_\chi}{\lambda_\phi}$, the program equations are

$$V_{pr} = \frac{a^4}{\lambda_\phi \phi_0^4} V = \frac{1}{4}\phi_{pr}^4 + \frac{1}{2} gl\ \phi_{pr}^2 \chi_{pr}^2 + \frac{1}{4} lcp\ \chi_{pr}^4 \tag{33}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \left(\phi_{pr}^2 + gl\ \chi_{pr}^2\right) \phi_{pr} \tag{34}$$

$$\frac{\partial V_{pr}}{\partial \chi_{pr}} = \left(gl\ \phi_{pr}^2 + lcp\ \chi_{pr}^2\right) \chi_{pr} \tag{35}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = 3\phi_{pr}^2 + gl\ \chi_{pr}^2 \tag{36}$$

$$\frac{\partial^2 V_{pr}}{\partial \chi_{pr}^2} = gl\ \phi_{pr}^2 + 3\ lcp\ \chi_{pr}^2 \tag{37}$$

6 SYMMBREAK

$$V = \frac{1}{4}\lambda\left(\phi^2 - v^2\right)^2 + \frac{1}{2}g^2\phi^2\chi^2. \tag{38}$$

During inflation the potential is $\frac{1}{4}\lambda\phi^4$ and all the rescalings (including the value of ϕ_0) are the same as for TWOFLD-LAMBDA.

The potential in terms of program variables is

$$V = \left(\frac{\phi_0}{a}\right)^4 \left[\frac{1}{4}\lambda\left(\phi_{pr}^2 - \frac{a^2v^2}{\phi_0^2}\right)^2 + \frac{1}{2}g^2\phi_{pr}^2\chi_{pr}^2\right] \tag{39}$$

Defining new variables $gl \equiv \frac{g^2}{\lambda}$ and $v_{pr} \equiv \frac{v}{\phi_0}$, the program equations are

$$V_{pr} = \frac{a^4}{\lambda\phi_0^4}V = \frac{1}{4}\left(\phi_{pr}^2 - a^2v_{pr}^2\right)^2 + \frac{1}{2}gl\ \phi_{pr}^2\chi_{pr}^2 \tag{40}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \left(\phi_{pr}^2 - a^2v_{pr}^2 + gl\ \chi_{pr}^2\right)\phi_{pr} \tag{41}$$

$$\frac{\partial V_{pr}}{\partial \chi_{pr}} = gl\ \phi_{pr}^2\chi_{pr} \tag{42}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = 3\phi_{pr}^2 - a^2v_{pr}^2 + gl\ \chi_{pr}^2 \tag{43}$$

$$\frac{\partial^2 V_{pr}}{\partial \chi_{pr}^2} = gl\ \phi_{pr}^2 \tag{44}$$

7 TWOFLDM

$$V = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 \chi^2. \tag{45}$$

The dominant potential term is $\frac{1}{2} m^2 \phi^2$ so

$$\beta = 2;\ cpl = m^2 \tag{46}$$

$$A = \frac{1}{\phi_0};\ B = m;\ r = \frac{3}{2};\ s = 0 \tag{47}$$

$$f_{pr} = \frac{a^{3/2}}{\phi_0} f;\ \vec{x}_{pr} = m\vec{x};\ dt_{pr} = m dt. \tag{48}$$

The potential in terms of program variables is

$$V = \frac{\phi_0^2}{a^3} \left(\frac{1}{2} m^2 \phi_{pr}^2 + \frac{1}{2} g^2 \frac{\phi_0^2}{a^3} \phi_{pr}^2 \chi_{pr}^2 \right) \tag{49}$$

Defining a new variable $gm \equiv \frac{g^2}{m^2}$, the program equations are

$$V_{pr} = \frac{a^3}{m^2 \phi_0^2} V = \frac{1}{2} \phi_{pr}^2 + \frac{1}{2} gm \frac{\phi_0^2}{a^3} \phi_{pr}^2 \chi_{pr}^2 \tag{50}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \left(1 + gm \frac{\phi_0^2}{a^3} \chi_{pr}^2 \right) \phi_{pr} \tag{51}$$

$$\frac{\partial V_{pr}}{\partial \chi_{pr}} = gm \frac{\phi_0^2}{a^3} \phi_{pr}^2 \chi_{pr} \tag{52}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = 1 + gm \frac{\phi_0^2}{a^3} \chi_{pr}^2 \tag{53}$$

$$\frac{\partial^2 V_{pr}}{\partial \chi_{pr}^2} = gm \frac{\phi_0^2}{a^3} \phi_{pr}^2. \tag{54}$$

The choice of ϕ_0 proceeds analogously to the TWOFLDLAMBDA case. Here

$$\frac{\partial}{\partial t_{pr}} \phi_{pr} \propto \left(\dot{f} + \frac{3}{2} \frac{\dot{a}}{a} f \right) \tag{55}$$

and the homogeneous field ϕ during and immediately after inflation obeys the equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + m^2 \phi \approx \ddot{\phi} + \sqrt{12\pi \left(\dot{\phi}^2 + m^2 \phi^2 \right)} \dot{\phi} + m^2 \phi = 0. \tag{56}$$

With the subsitution $t' \equiv mt$ this becomes

$$\ddot{\phi} + \sqrt{12\pi \left(\dot{\phi}^2 + \phi^2 \right)} \dot{\phi} + \phi = 0. \tag{57}$$

Solving this equation numerically we found that $\left(\dot{f} + \frac{3}{2} \frac{\dot{a}}{a} f \right)$ vanishes at $\phi \approx .193$, so we use this value for ϕ_0.

8 ONEFLDM

$$V = \frac{1}{2}m^2\phi^2. \tag{58}$$

This model is just the TWOFLDM model without the field χ. All rescalings, including the value of ϕ_0, are the same as for TWOFLDM. Since this model is for a free field it is primarily useful for testing purposes, e.g. testing new features in LATTICEEASY. The program equations are

$$V_{pr} = \frac{a^3}{m^2\phi_0^2}V = \frac{1}{2}\phi_{pr}^2 \tag{59}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \phi_{pr} \tag{60}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = 1 \tag{61}$$

9 NFLDM

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}h_i^2\chi^2\sigma_i^2. \tag{62}$$

Repeated indices are summed, so this model can include an arbitrary number of fields σ_i. This model is a generalization of the TWOFLDM model. In fact TWOFLDM is only included as a separate model because it provides a simple illustration of a model file. Running this model with $nflds = 2$ gives the exact same results. The dominant potential term is still $\frac{1}{2}m^2\phi^2$ and all the rescalings (including the value of ϕ_0) are the same as for TWOFLDM.

Defining new variables $gm \equiv \frac{g^2}{m^2}, hm_i \equiv \frac{h^2}{m^2}$, the program equations are

$$V_{pr} = \frac{a^3}{m^2\phi_0^2}V = \frac{1}{2}\phi_{pr}^2 + \frac{1}{2}gm\frac{\phi_0^2}{a^3}\phi_{pr}^2\chi_{pr}^2 + \frac{1}{2}hm_i\frac{\phi_0^2}{a^3}\chi_{pr}^2\sigma_{i,pr}^2 \tag{63}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \left(1 + gm\frac{\phi_0^2}{a^3}\chi_{pr}^2\right)\phi_{pr} \tag{64}$$

$$\frac{\partial V_{pr}}{\partial \chi_{pr}} = \frac{\phi_0^2}{a^3}\left(gm\ \phi_{pr}^2 + hm_i\ \sigma_{i,pr}^2\right)\chi_{pr} \tag{65}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{i,pr}} = hm_i\frac{\phi_0^2}{a^3}\chi_{pr}^2\sigma_{i,pr} \tag{66}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = 1 + gm\frac{\phi_0^2}{a^3}\chi_{pr}^2 \tag{67}$$

$$\frac{\partial^2 V_{pr}}{\partial \chi_{pr}^2} = \frac{\phi_0^2}{a^3}\left(gm\ \phi_{pr}^2 + hm_i\ \sigma_{i,pr}^2\right) \tag{68}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{i,pr}^2} = hm_i\frac{\phi_0^2}{a^3}\chi_{pr}^2. \tag{69}$$

10 TWOFLDHYBRID

$$V = \frac{1}{2} g^2 \phi^2 \sigma^2 + \frac{1}{4} \lambda \left(\sigma^2 - v^2\right)^2 . \tag{70}$$

This is the exact same potential as the SYMMBREAK model (with different names for the fields), but its interpretation is quite different. Now phi is the inflaton and the model is taken to be a hybrid model where the symmetry breaking term triggers the end of inflation. A realistic hybrid inflation model would also have a term like $1/2 m^2 \phi^2$ to drive slow roll inflation, but this term is typically negligible during reheating so we leave it out here. During the early stages of reheating when the field σ is falling to its symmetry breaking value $\sigma = \pm v$ the expansion of the universe is also negligible, so for this model we leave out the scale factor. Because the expansion is not included in these equations this model should only be run without expansion. Thus the model file for TWOFLDHYBRID includes a check that writes an error message and exits if the program isn't set to use no expansion.

The initial value of the inflaton is dictated by the symmetry breaking potential. There is a critical value $\phi_{cr} = \frac{\sqrt{\lambda} v}{g}$ such that for $\phi > \phi_{cr}$ the curvature in the σ direction is positive and σ remains fixed at zero, whereas for $\phi < \phi_{cr}$ the σ field will tend to fall to its minimum at $\pm v$. This ϕ mediated symmetry breaking is a generic feature of hybrid inflation models and it is the rapid fall of the σ field after ϕ passes ϕ_{cr} that ends inflation. Thus we take $\phi_0 = \phi_{cr} = \frac{\sqrt{\lambda} v}{g}$.

The determination of a dominant potential term for this model is complicated by the fact that the minimum is at $\phi = 0$, $\sigma = \pm v$. In terms of deviations from this minimum $\delta\sigma \equiv \sigma - v$ the potential near the minimum is given by $V \approx \lambda v^2 \delta\sigma^2$. This term would thus suggest

$$cpl = 2\lambda v^2, \tag{71}$$

but in practice the equations are simpler if we drop the 2, thus giving

$$A = \frac{g}{\sqrt{\lambda} v};\ B = \sqrt{\lambda} v \tag{72}$$

$$f_{pr} = \frac{g}{\sqrt{\lambda} v} f;\ x^\mu_{pr} = \sqrt{\lambda} v x^\mu \tag{73}$$

The potential in terms of program variables is

$$V = \frac{\lambda^2 v^4}{g^4} \left[\frac{1}{2} g^2 \phi_{pr}^2 \sigma_{pr}^2 + \frac{1}{4} \lambda \left(\sigma_{pr}^2 - \frac{g^2}{\lambda} \right)^2 \right]. \tag{74}$$

Defining a new variable $lg \equiv \frac{\lambda}{g^2}$, the program equations are

$$V_{pr} = \frac{g^2}{\lambda^2 v^4} V = \frac{1}{2} \phi_{pr}^2 \sigma_{pr}^2 + \frac{lg}{4} \left(\sigma_{pr}^2 - \frac{1}{lg} \right)^2 \tag{75}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \sigma_{pr}^2 \phi_{pr} \tag{76}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{pr}} = \left(\phi_{pr}^2 + lg\ \sigma_{pr}^2 - 1 \right) \sigma_{pr} \tag{77}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = \sigma_{pr}^2 \tag{78}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{pr}^2} = \phi_{pr}^2 + 3 lg\ \sigma_{pr}^2 - 1. \tag{79}$$

11 COMPLEXHYBRID

$$V = \frac{1}{2} g^2 \phi^2 |\sigma|^2 + \frac{1}{4} \lambda \left(|\sigma|^2 - v^2 \right)^2 . \tag{80}$$

This is the exact same potential as the TWOFLDYBRID model but with σ now a complex field. The rescalings are the same as for that model.

$$A = \frac{g}{\sqrt{\lambda} v}; \; B = \sqrt{\lambda} v \tag{81}$$

$$f_{pr} = \frac{g}{\sqrt{\lambda} v} f; \; x^\mu_{pr} = \sqrt{\lambda} v x^\mu \tag{82}$$

The potential in terms of program variables is

$$V = \frac{\lambda^2 v^4}{g^4} \left[\frac{1}{2} g^2 \phi_{pr}^2 |\sigma_{pr}|^2 + \frac{1}{4} \lambda \left(|\sigma_{pr}|^2 - \frac{g^2}{\lambda} \right)^2 \right] . \tag{83}$$

Defining a new variable $lg \equiv \frac{\lambda}{g^2}$, the program equations are

$$V_{pr} = \frac{g^2}{\lambda^2 v^4} V = \frac{1}{2} \phi_{pr}^2 |\sigma_{pr}|^2 + \frac{lg}{4} \left(|\sigma_{pr}|^2 - \frac{1}{lg} \right)^2 \tag{84}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = |\sigma_{pr}|^2 \phi_{pr} \tag{85}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{R,pr}} = \left(\phi_{pr}^2 + lg \, |\sigma_{pr}|^2 - 1 \right) \sigma_{R,pr} \tag{86}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{I,pr}} = \left(\phi_{pr}^2 + lg \, |\sigma_{pr}|^2 - 1 \right) \sigma_{I,pr} \tag{87}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = |\sigma_{pr}|^2 \tag{88}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{R,pr}^2} = \phi_{pr}^2 + 3 lg \, \sigma_{R,pr}^2 + lg \, \sigma_{I,pr}^2 - 1. \tag{89}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{I,pr}^2} = \phi_{pr}^2 + 3 lg \, \sigma_{I,pr}^2 + lg \, \sigma_{R,pr}^2 - 1. \tag{90}$$

12 FTERMHYBRID

Starting from the superpotential

$$W = \frac{\sqrt{\lambda}}{2}\Phi\left(4\bar{\Sigma}\Sigma - v^2\right) \tag{91}$$

we derive the potential

$$V = 4\lambda|\Phi|^2\left(|\Sigma|^2 + |\bar{\Sigma}|^2\right) + \frac{1}{4}\lambda|4\bar{\Sigma}\Sigma - v^2|^2. \tag{92}$$

With the convention $|F|^2 = \frac{1}{2}|f|^2 = \frac{1}{2}\left(f_R^2 + f_I^2\right)$ this becomes

$$V = \lambda\left[|\phi|^2\left(|\sigma|^2 + |\bar{\sigma}|^2\right) + |\sigma|^2|\bar{\sigma}|^2 - v^2\left(\sigma_R\bar{\sigma}_R - \sigma_I\bar{\sigma}_I\right) + \frac{1}{4}v^4\right] \tag{93}$$

As in the TWOFLDHYBRID model we neglect expansion. The initial value of the inflaton is given by the point where the curvature of the Σ fields becomes negative, namely $\phi_0 = \frac{v}{\sqrt{2}}$. For the spacetime rescaling we use the same choice as we did for the TWOFLDHYBRID model; see the discussion there. So

$$A = \frac{\sqrt{2}}{v};\ B = \sqrt{\lambda}v \tag{94}$$

$$f_{pr} = \frac{\sqrt{2}}{v}f;\ x^\mu_{pr} = \sqrt{\lambda}vx^\mu \tag{95}$$

The potential in terms of program variables is

$$V = \frac{\lambda v^4}{4}\left[|\phi_{pr}|^2\left(|\sigma_{pr}|^2 + |\bar{\sigma}_{pr}|^2\right) + |\sigma_{pr}|^2|\bar{\sigma}_{pr}|^2 - 2\left(\sigma_{R,pr}\bar{\sigma}_{R,pr} - \sigma_{I,pr}\bar{\sigma}_{I,pr}\right) + 1\right] \tag{96}$$

$$V_{pr} = \frac{2}{\lambda v^4}V = \frac{1}{2}|\phi_{pr}|^2\left(|\sigma_{pr}|^2 + |\bar{\sigma}_{pr}|^2\right) + \frac{1}{2}|\sigma_{pr}|^2|\bar{\sigma}_{pr}|^2 - \left(\sigma_{R,pr}\bar{\sigma}_{R,pr} - \sigma_{I,pr}\bar{\sigma}_{I,pr}\right) + \frac{1}{2} \tag{97}$$

$$\frac{\partial V_{pr}}{\partial\phi_{R,pr}} = \left(|\sigma_{pr}|^2 + |\bar{\sigma}_{pr}|^2\right)\phi_{R,pr} \tag{98}$$

$$\frac{\partial V_{pr}}{\partial\phi_{I,pr}} = \left(|\sigma_{pr}|^2 + |\bar{\sigma}_{pr}|^2\right)\phi_{I,pr} \tag{99}$$

$$\frac{\partial V_{pr}}{\partial\sigma_{R,pr}} = \left(|\phi_{pr}|^2 + |\bar{\sigma}_{pr}|^2\right)\sigma_{R,pr} - \bar{\sigma}_{R,pr} \tag{100}$$

$$\frac{\partial V_{pr}}{\partial\sigma_{I,pr}} = \left(|\phi_{pr}|^2 + |\bar{\sigma}_{pr}|^2\right)\sigma_{I,pr} + \bar{\sigma}_{I,pr} \tag{101}$$

$$\frac{\partial V_{pr}}{\partial\bar{\sigma}_{R,pr}} = \left(|\phi_{pr}|^2 + |\sigma_{pr}|^2\right)\bar{\sigma}_{R,pr} - \sigma_{R,pr} \tag{102}$$

$$\frac{\partial V_{pr}}{\partial\bar{\sigma}_{I,pr}} = \left(|\phi_{pr}|^2 + |\sigma_{pr}|^2\right)\bar{\sigma}_{I,pr} + \sigma_{I,pr} \tag{103}$$

$$\frac{\partial^2 V_{pr}}{\partial\phi^2_{R,pr}} = \frac{\partial^2 V_{pr}}{\partial\phi^2_{I,pr}} = |\sigma_{pr}|^2 + |\bar{\sigma}_{pr}|^2 \tag{104}$$

$$\frac{\partial^2 V_{pr}}{\partial\sigma^2_{R,pr}} = \frac{\partial^2 V_{pr}}{\partial\sigma^2_{I,pr}} = |\phi_{pr}|^2 + |\bar{\sigma}_{pr}|^2 \tag{105}$$

$$\frac{\partial^2 V_{pr}}{\partial\bar{\sigma}^2_{R,pr}} = \frac{\partial^2 V_{pr}}{\partial\bar{\sigma}^2_{I,pr}} = |\phi_{pr}|^2 + |\sigma_{pr}|^2 \tag{106}$$

13 DTERMHYBRID

Starting from the superpotential

$$W = \sqrt{2} g \Phi \Sigma_+ \Sigma_- \tag{107}$$

and the D-term

$$D = \frac{\sqrt{\lambda}}{2} \left(2|\Sigma_+|^2 - 2|\Sigma_-|^2 - v^2 \right) \tag{108}$$

we derive the potential

$$V = 2g^2 \left[|\Phi|^2 \left(|\Sigma_+|^2 + |\Sigma_-|^2 \right) + |\Sigma_+|^2 |\Sigma_-|^2 \right] + \frac{1}{4} \lambda \left(2|\Sigma_+|^2 - 2|\Sigma_-|^2 - v^2 \right)^2 \tag{109}$$

With the convention $|F|^2 = \frac{1}{2}|f|^2 = \frac{1}{2}\left(f_R^2 + f_I^2\right)$ this becomes

$$V = \frac{1}{2} g^2 \left[|\phi|^2 \left(|\sigma_+|^2 + |\sigma_-|^2 \right) + |\sigma_+|^2 |\sigma_-|^2 \right] + \frac{1}{4} \lambda \left(|\sigma_+|^2 - |\sigma_-|^2 - v^2 \right)^2 \tag{110}$$

As in the TWOFLDHYBRID model we neglect expansion. The initial value of the inflaton is given by the point where the curvature of the Σ fields becomes negative, namely $\phi_0 = \frac{\sqrt{\lambda} v}{g}$. For the spacetime rescaling we use the same choice as we did for the TWOFLDHYBRID model; see the discussion there. So

$$A = \frac{g}{\sqrt{\lambda} v}; \; B = \sqrt{\lambda} v \tag{111}$$

$$f_{pr} = \frac{g}{\sqrt{\lambda} v} f; \; x^\mu_{pr} = \sqrt{\lambda} v x^\mu \tag{112}$$

The potential in terms of program variables is

$$V = \frac{\lambda^2 v^4}{g^4} \left[\frac{1}{2} g^2 \left[|\phi_{pr}|^2 \left(|\sigma_{+,pr}|^2 + |\sigma_{-,pr}|^2 \right) + |\sigma_{+,pr}|^2 |\sigma_{-,pr}|^2 \right] + \frac{1}{4} \lambda \left(|\sigma_{+,pr}|^2 - |\sigma_{-,pr}|^2 - \frac{g^2}{\lambda} \right)^2 \right] \tag{113}$$

Defining a new variable $lg \equiv \frac{\lambda}{g^2}$, the program equations are

$$V_{pr} = \frac{g^2}{\lambda^2 v^4} V = \frac{1}{2} \left[|\phi_{pr}|^2 \left(|\sigma_{+,pr}|^2 + |\sigma_{-,pr}|^2 \right) + |\sigma_{+,pr}|^2 |\sigma_{-,pr}|^2 \right] + \frac{1}{4} lg \left(|\sigma_{+,pr}|^2 - |\sigma_{-,pr}|^2 - \frac{1}{lg} \right)^2 \tag{114}$$

$$\frac{\partial V_{pr}}{\partial \phi_{R,pr}} = \left(|\sigma_{+,pr}|^2 + |\sigma_{-,pr}|^2 \right) \phi_{R,pr} \tag{115}$$

$$\frac{\partial V_{pr}}{\partial \phi_{I,pr}} = \left(|\sigma_{+,pr}|^2 + |\sigma_{-,pr}|^2 \right) \phi_{I,pr} \tag{116}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{+R,pr}} = \left(|\phi_{pr}|^2 + (1 - lg)|\sigma_{-,pr}|^2 + lg|\sigma_{+,pr}|^2 - 1 \right) \sigma_{+R,pr} \tag{117}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{+I,pr}} = \left(|\phi_{pr}|^2 + (1 - lg)|\sigma_{-,pr}|^2 + lg|\sigma_{+,pr}|^2 - 1 \right) \sigma_{+I,pr} \tag{118}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{-R,pr}} = \left(|\phi_{pr}|^2 + (1 - lg)|\sigma_{+,pr}|^2 + lg|\sigma_{-,pr}|^2 + 1 \right) \sigma_{-R,pr} \tag{119}$$

$$\frac{\partial V_{pr}}{\partial \sigma_{-I,pr}} = \left(|\phi_{pr}|^2 + (1 - lg)|\sigma_{+,pr}|^2 + lg|\sigma_{-,pr}|^2 + 1 \right) \sigma_{-I,pr} \tag{120}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{R,pr}^2} = \frac{\partial^2 V_{pr}}{\partial \phi_{I,pr}^2} = |\sigma_{+,pr}|^2 + |\sigma_{-,pr}|^2 \tag{121}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{+R,pr}^2} = |\phi_{pr}|^2 + (1 - lg)|\sigma_{-,pr}|^2 + lg \left(\sigma_{+I,pr}^2 + 3\sigma_{+R,pr}^2 \right) - 1 \tag{122}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{+I,pr}^2} = |\phi_{pr}|^2 + (1-lg)|\sigma_{-,pr}|^2 + lg\left(\sigma_{+R,pr}^2 + 3\sigma_{I,pr^2}\right) - 1 \tag{123}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{-R,pr}^2} = |\phi_{pr}|^2 + (1-lg)|\sigma_{+,pr}|^2 + lg\left(\sigma_{-I,pr}^2 + 3\sigma_{-R,pr}^2\right) + 1 \tag{124}$$

$$\frac{\partial^2 V_{pr}}{\partial \sigma_{-I,pr}^2} = |\phi_{pr}|^2 + (1-lg)|\sigma_{+,pr}|^2 + lg\left(\sigma_{-R,pr}^2 + 3\sigma_{-I,pr}^2\right) + 1 \tag{125}$$

14 QINFLATION

$$V = V_0 e^{-2v/|\phi|} + \frac{1}{4}\lambda|\phi|^4 \tag{126}$$

where $|\phi| = \sqrt{\phi_R^2 + \phi_I^2}$. Because expansion is not included in these equations this model should only be run without expansion. Thus the model file for QINFLATION includes a check that writes an error message and exits if the program isn't set to use no expansion.

The initial value of the inflaton is $\frac{\sqrt{v}}{\pi^{1/4}}$. We define a new variable $v_{pr} \equiv 2\pi^{1/4}\sqrt{v}$. We set the rescalings directly as

$$A = \frac{1}{\phi_0} = \frac{\pi^{1/4}}{\sqrt{v}} = \frac{2\sqrt{\pi}}{v_{pr}};\ B = 2\sqrt{\frac{\pi V_0}{v_{pr}}} \tag{127}$$

$$f_{pr} = \frac{2\sqrt{\pi}}{v_{pr}} f;\ x_{pr}^\mu = 2\sqrt{\frac{\pi V_0}{v_{pr}}} x^\mu \tag{128}$$

The potential in terms of program variables is

$$V = V_0 \exp\left[\frac{-v_{pr}^2}{2\sqrt{\pi}|\phi|}\right] + \frac{1}{4}\lambda|\phi|^4 = V_0 e^{-v_{pr}/|\phi_{pr}|} + \frac{v_{pr}^4\lambda}{64\pi^2}|\phi_{pr}|^4 \tag{129}$$

The program potential is

$$V_{pr} = \frac{1}{v_{pr}V_0} V = \frac{1}{v_{pr}} e^{-v_{pr}/|\phi_{pr}|} + \frac{v_{pr}^3\lambda}{64\pi^2 V_0}|\phi_{pr}|^4 \tag{130}$$

Defininining a new variable $\lambda_{pr} \equiv \frac{v_{pr}^3\lambda}{16\pi^2 V_0}$ the program equations are

$$V_{pr} = \frac{1}{v_{pr}} e^{-v_{pr}/|\phi_{pr}|} + \frac{1}{4}\lambda_{pr}|\phi_{pr}|^4 \tag{131}$$

$$\frac{\partial V_{pr}}{\partial \phi_{R,pr}} = |\phi_{pr}|^{-3} e^{-v_{pr}/|\phi_{pr}|}\phi_{R,pr} + \lambda_{pr}|\phi_{pr}|^2\phi_{R,pr} \tag{132}$$

$$\frac{\partial V_{pr}}{\partial \phi_{I,pr}} = |\phi_{pr}|^{-3} e^{-v_{pr}/|\phi_{pr}|}\phi_{I,pr} + \lambda_{pr}|\phi_{pr}|^2\phi_{I,pr} \tag{133}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{R,pr}^2} = \left[|\phi_{pr}|^{-3} - 3\phi_{R,pr}^2|\phi_{pr}|^{-5} + v_{pr}\phi_{R,pr}^2|\phi_{pr}|^{-6}\right] e^{-v_{pr}/|\phi_{pr}|} + 3\lambda_{pr}\phi_{R,pr}^2 + \lambda_{pr}\phi_{I,pr}^2 \tag{134}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{I,pr}^2} = \left[|\phi_{pr}|^{-3} - 3\phi_{I,pr}^2|\phi_{pr}|^{-5} + v_{pr}\phi_{I,pr}^2|\phi_{pr}|^{-6}\right] e^{-v_{pr}/|\phi_{pr}|} + 3\lambda_{pr}\phi_{I,pr}^2 + \lambda_{pr}\phi_{R,pr}^2 \tag{135}$$

15 PLAINSYMMBREAK

$$V = \frac{1}{4}\lambda\left(\phi^2 - v^2\right)^2. \tag{136}$$

This is a simplified version of the SYMMBREAK model. Here there is no field other than ϕ, which will be taken to start at the top of the potential hill at $\phi = 0$. Expansion is neglected. The rescaling parameters are set to

$$A = \frac{1}{v};\ B = \sqrt{\lambda}v \tag{137}$$

$$f_{pr} = \frac{1}{v}f;\ x^\mu_{pr} = \sqrt{\lambda}vx^\mu \tag{138}$$

The potential in terms of program variables is

$$V = \frac{\lambda v^4}{4}\left(\phi_{pr}^2 - 1\right)^2 \tag{139}$$

and the program equations are

$$V_{pr} = \frac{1}{\lambda v^4}V = \frac{1}{4}\left(\phi_{pr}^2 - 1\right)^2 \tag{140}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \left(\phi_{pr}^2 - 1\right)\phi_{pr} \tag{141}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = 3\phi_{pr}^2 - 1 \tag{142}$$

16 COMPLEXSYMMBREAK

$$V = \lambda \left(|\Phi|^2 - \frac{1}{2}v^2 \right)^2 . \tag{143}$$

This is a complex version of the PLAINSYMMBREAK model. There is no field other than Φ, which will be taken to start at the top of the potential hill at $\Phi = 0$. Expansion is neglected. Using the standard redefinition $\phi = \phi_R + i\phi_I \equiv \sqrt{2}\Phi$ the potential becomes

$$V = \frac{1}{4}\lambda \left(|\phi|^2 - v^2 \right)^2 . \tag{144}$$

The rescaling parameters are set to

$$A = \frac{1}{v};\ B = \sqrt{\lambda}v \tag{145}$$

$$f_{pr} = \frac{1}{v}f;\ x^\mu_{pr} = \sqrt{\lambda}vx^\mu \tag{146}$$

The potential in terms of program variables is

$$V = \frac{\lambda v^4}{4} \left(|\phi_{pr}|^2 - 1 \right)^2 \tag{147}$$

and the program equations are

$$V_{pr} = \frac{1}{\lambda v^4}V = \frac{1}{4} \left(|\phi_{pr}|^2 - 1 \right)^2 \tag{148}$$

$$\frac{\partial V_{pr}}{\partial \phi_{R,pr}} = \left(|\phi_{pr}|^2 - 1 \right) \phi_{R,pr} \tag{149}$$

$$\frac{\partial V_{pr}}{\partial \phi_{I,pr}} = \left(|\phi_{pr}|^2 - 1 \right) \phi_{I,pr} \tag{150}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi^2_{R,pr}} = 3\phi^2_{R,pr} + \phi^2_{I,pr} - 1 \tag{151}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi^2_{I,pr}} = 3\phi^2_{I,pr} + \phi^2_{R,pr} - 1 \tag{152}$$

17 CUBICSYMMBREAK

$$V = \frac{1}{4}\lambda\phi^4 - \frac{1}{3}\lambda v\phi^3 + \frac{1}{12}\lambda v^4. \tag{153}$$

This is not technically a symmetry breaking potential since there is no symmetry to be broken, however it models the basic features of some symmetric models including F-Term hybrid inflation and some new inflationary models. For F-Term inflation expansion can be neglected since the Hubble constant is small at the end of inflation. For new inflation expansion should generally be included. The model file for this potential includes two constants finit and fdinit corresponding to the values of ϕ and $\dot{\phi}$ a the end of new inflation.

The rescalings are

$$A = \frac{1}{v};\ B = \sqrt{\lambda}v;\ r = \frac{3}{2};\ s = 0 \tag{154}$$

$$f_{pr} = a^{3/2}\frac{1}{v}f;\ \vec{x}_{pr} = \sqrt{\lambda}v\vec{x};\ dt_{pr} = \sqrt{\lambda}vdt. \tag{155}$$

The potential in terms of program variables is

$$V = \lambda v^4\left(\frac{1}{4}a^{-6}\phi_{pr}^4 - \frac{1}{3}a^{-9/2}\phi_{pr}^3 + \frac{1}{12}\right) \tag{156}$$

and the program equations are

$$V_{pr} = \frac{1}{\lambda v^4}a^3V = \frac{1}{4}a^{-3}\phi_{pr}^4 - \frac{1}{3}a^{-3/2}\phi_{pr}^3 + \frac{1}{12}a^3 \tag{157}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = \left(a^{-3}\phi_{pr} - a^{-3/2}\right)\phi_{pr}^2 \tag{158}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = \left(3a^{-3}\phi_{pr} - 2a^{-3/2}\right)\phi_{pr} \tag{159}$$

18 COLEMANWEINBERG

$$V = \frac{1}{4}\lambda\phi^4 \left(ln \left(\frac{|\phi|}{v} \right) - \frac{1}{4} \right) + \frac{1}{16}\lambda v^4. \tag{160}$$

This is the low temperature limit of the Coleman-Weinberg effective potential used in new inflation. I've changed the argument of the logarithm to an absolute value so that the theory can be well defined for all values of ϕ.

The rescalings are

$$A = \frac{1}{v};\ B = \sqrt{\lambda}v;\ r = \frac{3}{2};\ s = 0 \tag{161}$$

$$f_{pr} = a^{3/2}\frac{1}{v}f;\ \vec{x}_{pr} = \sqrt{\lambda}v\vec{x};\ dt_{pr} = \sqrt{\lambda}v dt. \tag{162}$$

The potential in terms of program variables is

$$V = \lambda v^4 \left[\frac{1}{4} a^{-6} \phi_{pr}^4 \left(ln \left(a^{-3/2}|\phi_{pr}| \right) - \frac{1}{4} \right) + \frac{1}{16} \right] \tag{163}$$

and the program equations are

$$V_{pr} = \frac{1}{\lambda v^4} a^3 V = \frac{1}{4} a^{-3} \phi_{pr}^4 \left(ln \left(a^{-3/2}|\phi_{pr}| \right) - \frac{1}{4} \right) + \frac{1}{16} a^3 \tag{164}$$

$$\frac{\partial V_{pr}}{\partial \phi_{pr}} = a^{-3} \phi_{pr}^3 ln \left(a^{-3/2}|\phi_{pr}| \right) \tag{165}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{pr}^2} = a^{-3} \left(3\ ln \left(a^{-3/2}|\phi_{pr}| \right) + 1 \right) \phi_{pr}^2 \tag{166}$$

19 QUARTICSYMMBREAK

$$V = -\frac{1}{4}\lambda\phi^4 + \frac{1}{6v^2}\lambda\phi^6 \tag{167}$$

Symmetry breaking potential with a negative quartic term.

The rescalings are

$$A = \frac{1}{v};\ B = \sqrt{\lambda}v;\ r = \frac{3}{2};\ s = 0 \tag{168}$$

$$f_{pr} = a^{3/2}\frac{1}{v}f;\ \vec{x}_{pr} = \sqrt{\lambda}v\vec{x};\ dt_{pr} = \sqrt{\lambda}vdt. \tag{169}$$

The potential in terms of program variables is

$$V = \lambda v^4\left(-\frac{1}{4}a^{-6}\phi_{pr}^4 + \frac{1}{6}a^{-9}\phi_{pr}^6\right) \tag{170}$$

and the program equations are

$$V_{pr} = \frac{1}{\lambda v^4}a^3V = -\frac{1}{4}a^{-3}\phi_{pr}^4 + \frac{1}{6}a^{-6}\phi_{pr}^6 \tag{171}$$

$$\frac{\partial V_{pr}}{\partial\phi_{pr}} = -a^{-3}\phi_{pr}^3 + a^{-6}\phi_{pr}^5 \tag{172}$$

$$\frac{\partial^2 V_{pr}}{\partial\phi_{pr}^2} = -3a^{-3}\phi_{pr}^2 + 5a^{-6}\phi_{pr}^4. \tag{173}$$

20 TKACHEVSTRINGS

$$V = \frac{1}{4}\lambda\left(|\phi|^2 - v^2\right)^2. \tag{174}$$

This is a complex version of the SYMMBREAK model, only without the extra field(s) χ. There is no field other than ϕ, which is initially set to $\phi = \phi_0 = .35$. The rescaling parameters are set to

$$A = \frac{1}{\phi_0};\; B = \sqrt{\lambda}\phi_0;\; r = 1;\; s = -1 \tag{175}$$

$$f_{pr} = \frac{a}{\phi_0} f;\; \vec{x}_{pr} = \sqrt{\lambda} v \vec{x};\; dt_{pr} = \frac{\sqrt{\lambda} v}{a} dt \tag{176}$$

The potential in terms of program variables is

$$V = \left(\frac{\phi_0}{a}\right)^4 \frac{1}{4}\lambda\left(|\phi_{pr}|^2 - \left(\frac{a^2}{\phi_0^2}\right)^2 v^2\right)^2. \tag{177}$$

Defining a new variable $v_{pr} \equiv \frac{v}{\phi_0}$ the program equations are

$$V_{pr} = \frac{a^4}{\lambda\phi_0^4} V = \frac{1}{4}\left(|\phi_{pr}|^2 - a^2 v_{pr}^2\right)^2 \tag{178}$$

$$\frac{\partial V_{pr}}{\partial \phi_{R,pr}} = \left(|\phi_{pr}|^2 - a^2 v_{pr}^2\right)\phi_{R,pr} \tag{179}$$

$$\frac{\partial V_{pr}}{\partial \phi_{I,pr}} = \left(|\phi_{pr}|^2 - a^2 v_{pr}^2\right)\phi_{I,pr} \tag{180}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{R,pr}^2} = 3\phi_{R,pr}^2 + \phi_{I,pr}^2 - a^2 v_{pr}^2 \tag{181}$$

$$\frac{\partial^2 V_{pr}}{\partial \phi_{I,pr}^2} = 3\phi_{I,pr}^2 + \phi_{R,pr}^2 - a^2 v_{pr}^2 \tag{182}$$