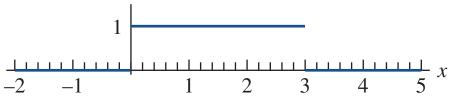
APPENDIX L

Answers to "Check Yourself" in Exercises

1. The force is towards the equilibrium point: left if the object is on the right side and right if it's on the left. The resulting motion will be an oscillation.
2. $d^2x/dt^2 = -(k/m)x$. In words, we might read this equation as follows: "The position function $x(t)$ has the property that when you take its second derivative, you get the same function you started with, multiplied by $-k/m$." As a quick check, the left side of this equation has units m/s^2 and the right side has $N/m/kg\ m$, or N/kg. Since a Newton is a kg m/s^2, this becomes m/s^2, so the units match on the two sides.
3. One possible answer is $y(x) = 3x^2 + 2$.
4. $dy/dx = y$
5. $y = \pm e^{x^3/3+C}$.
6. $u_1(x) = e^{-x^2}$, $u_2(x) = e^{2x^2}$
7. $dR/dt = 5R - 10F$
8. $f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$. This formula should not come as any great surprise. Since $f'(x_0)$ represents the slope (or "rise over run"), we multiply it by Δx (the "run") to estimate how much the function has gone *up* from its original value of $f(x_0)$.
9. 605.
10. If you plug $a_1 = 5$, $r = 3$, and $n = 5$ into your formula, you reproduce our original series. Your formula should therefore yield the answer 605. If it doesn't, figure out what went wrong!
11. $1/2$
12. $x = Ae^{-10t} + Be^{-4t}$
13. $x = Ae^{\left(-2+\sqrt{-36}\right)t} + Be^{\left(-2-\sqrt{-36}\right)t}$.
14. 34
15. $B = i$
16. $\dfrac{1}{1+2i}e^{(1+2i)x} + C$
17. $y = 1/2$
18. A line has zero concavity. Because $\partial^2 y/\partial x^2 = 0$ the equation predicts $\partial^2 y/\partial t^2 = 0$: no acceleration. Because it started at rest and is not accelerating, the string will never move.
19. positive
20. $y(50) = 130$
21. $-\infty$
22. down, 2
23. $\dfrac{\partial f}{\partial x}u_x + \dfrac{\partial f}{\partial y}u_y = 1\left(\dfrac{1}{\sqrt{1+c^2}}\right) + 2\left(\dfrac{c}{\sqrt{1+c^2}}\right) = \dfrac{1+2c}{\sqrt{1+c^2}}$
24. $a = \dfrac{\partial f}{\partial x}\left(x_0, y_0\right)$ (We'll leave b and c for you to determine.)

25. $V = -\dfrac{2+\sqrt{2}}{4}\dfrac{GM}{d}$. (Even if your answer is correct it may take some simplification to make it look like ours. You can easily check your answer by punching it into a calculator. Ours comes out to $-0.85GM/d$.)
26. 56 miles
27. $M = \int_0^W (kx)(Hdx) = (1/2)kW^2H$. Since kx has units of mass per distance squared, k must have units of mass per distance cubed, so this answer does have units of mass.
28. $M = (1/4)qW^2H^2$
29. $m = 2kR^5/15$. Since σ is mass over distance squared k must have units of mass over distance to the fifth, so the units are correct.
30. $\rho = 5$, $\phi = \tan^{-1}(4/3)$, $z = 10$
31. $r = \sqrt{125}$, $\theta = \cos^{-1}(10/\sqrt{125})$, $\phi = \tan^{-1}(4/3)$
32. $\left(2\hat{i} + 3\hat{j}\right) \cdot \left(4\hat{i} + 24\hat{j}\right) = 80$
33. $19,544/15 \approx 1303$
34. You should have graphed the curve $y = x^2$.
35. For $v = 2$ you get the parabola $y = (x-4)^2$ on the plane $z = 4$.
36. The key is the surface area of the top. The water accumulated is the water that has passed through the top of the bucket.
37. *(rate of rainfall)* × *(top area of bucket)*. This might be written $W = rA$.
38. 180 and 122,000
39. Vector $\vec{v}_2 = \vec{A} - 2\vec{B}$. Count yourself correct if you had something reasonably close to that.
40. $820,000,008$ and $6,800$
41. $w = 3$. We'll leave it to you to find the others. (Of course the real way to check yourself is to see if $\mathbf{AB} = \mathbf{I}$!)
42. $x = 49/8$, $y = -7/4$
43. Matrix **B** stretches any shape by a factor of 3 in the x-direction and a factor of 5 in the y-direction.
44. $x_f = \rho_0 \cos(\phi_0 + \Delta\phi)$, $y_f = \rho_0 \sin(\phi_0 + \Delta\phi)$
45. $\mathbf{C} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$
46. $\mathbf{M} = \mathbf{C}^{-1}\mathbf{M}'\mathbf{C} = \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix}$
47. $(3, 2)$
48. $\hat{i} + 9\hat{j}$
49. the x-component
50. The depth increases as you walk. The farther you go, the faster it increases.
51. The first leaf moves in a line at constant speed. The second leaf begins with a velocity of $\vec{v} = \hat{i} + 3\hat{j}$ (speed of $\sqrt{10}$) and gradually turns to the right. It keeps moving in both the x- and y-directions forever, but its direction gets closer and closer to horizontal. The third leaf's long-term behavior (in the next part) is quite different.
52. It will roll to the left, gaining speed as it approaches A and then losing speed as it continues to the left. It will come to a stop when it reaches the same height it started at. Then it will begin rolling back to the right. Over time it will oscillate between these two points (with different x-values but the same $V(x)$-value) forever.
53. positive x, no y-movement
54. neither
55. $\hat{p}_x = 6/\sqrt{45}$, $\hat{p}_y = 3/\sqrt{45}$
56. $-1/2$
57. $2\pi/19$ Earth years
58. By eye, it's roughly $2\cos(3x) + 0.3\cos(20x)$

59. $4\pi/5$
60. $2i/(3\pi)$
61. $\pi/500, 2\pi/500, 3\pi/500 \ldots$
62. $d^2x/dt^2 + 6(dx/dt) + 9x = 0$
63. $A(x)$
64. $A\sin(kx) + B\cos(kx)$, $A\sin(kx+\phi)$, $Ae^{ikx} + B\sin(kx)$, $Ae^{ikx} + Be^{-ikx}$
65. $du/dt = dx/dt + 1$
66. $du/dt = u^2$
67. $xu''(x) - u'(x) = 0$
68. 

69. $z = e^x \sin y$.
70. $\pi/15$.
71. One of your equations should be rewriteable as $X''(x) = PX(x)$.
72. Your solutions in Parts 5, 6, and 7 should have been exponential, linear, and sinusoidal, respectively. Since an exponential or linear function cannot reach two zeros, you should have concluded that P must be negative. We will now replace P with $-k^2$, which allows for all possible negative values.
73. $X(x) = A\sin(n\pi x/w)$
74. $z(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin(kx)\sin(py)\left[C_{nm}\sin\left(v\sqrt{p^2+k^2}\,t\right) + D_{nm}\cos\left(v\sqrt{p^2+k^2}\,t\right)\right]$ where $k = n\pi/w$ and $p = m\pi/h$.
75. $\dfrac{R''(\rho)}{R(\rho)} + \dfrac{1}{\rho}\dfrac{R'(\rho)}{R(\rho)} = \dfrac{-Z''(z)}{Z(z)}$. Both sides of your equation must now equal a constant. Because the boundary conditions on $R(\rho)$ are homogeneous, we consider the ODE for $R(\rho)$ first to determine the sign of the separation constant.
76. $V(\rho,z) = \sum_{n=1}^{\infty} A_n J_0\left(\alpha_{0,n}\rho/a\right)\left(e^{(\alpha_{0,n}/a)z} - e^{-(\alpha_{0,n}/a)z}\right)$
77. $\sum_{n=1}^{\infty} b_n'(t)\sin\left(\dfrac{n\pi}{L}x\right) = \sum_{n=1}^{\infty} -\dfrac{\alpha b_n(t)n^2\pi^2}{L^2}\sin\left(\dfrac{n\pi}{L}x\right)$
78. $(\partial\mathcal{F}[u]/\partial t) = -\alpha p^2\mathcal{F}[u]$
79. $R(\rho) = CJ_0(k\rho) + DY_0(k\rho)$
80. $k = \alpha_{0,n}/a$, where $\alpha_{0,n}$ is the nth zero of J_0.
81. $z = J_0\left(\dfrac{\alpha_{0,n}}{a}\rho\right)\left[A\sin\left(\dfrac{\alpha_{0,n}}{a}vt\right) + B\cos\left(\dfrac{\alpha_{0,n}}{a}vt\right)\right]$
82. $f(\rho) = \sum_{n=1}^{\infty} B_n J_0\left(\frac{\alpha_{0,n}}{a}\rho\right)$
83. $dy/dx = c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + 5c_5x^4 + \ldots$
84. $2c_2 = c_1 + 1$
85. $c_{n+2} = \dfrac{n^2+n-\mu}{(n+1)(n+2)}c_n$
86. $r = -3$, $r = -4$
87. $R(r) = Aj_0\left(\dfrac{\sqrt{2mE}}{\hbar}r\right) + By_0\left(\dfrac{\sqrt{2mE}}{\hbar}r\right)$
88. $\cos x + x\sin x$
89. $D^2 - x^2 - 1$
90. $T = 1 - (2/\pi)x$

91. $T = (2/\pi)\phi$

92. $\ln z = \ln|z| + i(\phi + 2\pi n)$, where n is any integer.

93. $z = 5 + i$, $\Delta z = i$, $f(z + \Delta z) = (5 + i)^2 = 24 + 10i$, $\Delta f = -1 + 10i$, so $\Delta f/\Delta z = 10 + i$.

94. $2i$

95. The line segment from $z = 0$ to $z = 2$ maps to a line segment from $f = 0$ to $f = 4$, also on the positive real axis.

1000. $Q(t) = Ae^{-100{,}000\,t} + Be^{-1000\,t}$

1001. $Q(t) = Ae^{\left(-500{,}000+500{,}000\sqrt{-1}\right)t} + Be^{\left(-500{,}000-500{,}000\sqrt{-1}\right)t}$

1002. D

1003. $\dfrac{dP}{dx} = -\dfrac{8\mu RTw}{\pi r^4 M}\dfrac{1}{P}$